Kinetic Formulations of the compressible Euler equation with spherical symmetry

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1. Introduction

One of the most important nonlinear PDEs in both physics and mathematics is the compressible Euler equation for an isentropic gas. It is given by, in $\mathbf{R}^3$,

$$(1.1)\qquad \begin{cases} \rho_t + \sum_{j=1}^{3} \dfrac{\partial}{\partial x_j}(\rho u_j) = 0 \ , \\ (\rho u_i)_t + \sum_{j=1}^{3} \dfrac{\partial}{\partial x_j}(\rho u_i u_j + \delta_{ij} p) = 0 \ , \ \ (i = 1, 2, 3) \ . \end{cases}$$

with the equation of state

$$(1.2)\qquad p \ = \ C\rho^{\gamma} \ .$$

Here, the density ρ, velocity $\mathbf{u} = {}^t(u_1, u_2, u_3)$ and pressure p are functions of $x \in \mathbf{R}^n$ and $t \geq 0$ and $C > 0$ and $\gamma \geq 1$ are given constants. Besides its physical meaning, it is well-known as a typical example of conservation laws and nonlinear hyperbolic system. In fact, many interesting theories have been discovered by studying this equation.

Let us briefly recall the history of (1.1). In one dimensional case, T. Nishida [18] has first discovered global weak solution by using Glimm's Theory [5] for the case $\gamma = 1$. The key point of his success is deriving uniform estimates of the total variation of approximate solutions by considering the variation of Riemann invariants. But unfortunately, for the case $\gamma > 1$, this method can not be applied. Indeed, we can not obtain uniform estimates of the total variation in this case. This lack of uniform estimates of the total variation caused, in fact, many difficulties and the existence of global weak solution had been an important open problem. In 1982, this problem is finally solved by R. DiPerna [4]. By only using uniform L^∞ estimates, he has showed the existence of global weak solution by applying compensated compactness (L. Tartar[20]) and the notion of entropy(P. D. Lax [7]) for the case $\gamma = 1 + \frac{2}{n+2}$ (n : integer and odd). Later, Ding, Chen et Luo have extended this result for the case $1 < \gamma \leq \frac{5}{3}$ by using Lax-Friedrichs scheme ([2]).

In 1992 P.L.Lions, B. Perthame and E. Tadmor [8] proposed the so-called kinetic formulation which is based on the Lax's notion of entropy. In [8] they showed the existence of global weak solution in more clear way by using this method for the case $\gamma \geq 3$. In [9] they extended this result to the case $\gamma < 3$.

But for the multi-dimensional case, only local classical solutions are known to exist.(see [10]). Only for the spherically symmetric case, there are several results for the weak solutions. Assuming that solutions are of the form

$$\rho = \rho(t, |x|), \ \ \vec{u} = \frac{x}{|x|} \cdot u(t, |x|). \tag{1.3}$$

Then, denoting $r = |x|$, (1.1) becomes

$$\begin{cases} \dfrac{\partial}{\partial t}\rho + \dfrac{\partial}{\partial r}(\rho u) + \dfrac{2}{r}\rho u = 0\,, \\ \dfrac{\partial}{\partial t}(\rho u) + \dfrac{\partial}{\partial r}\left(\rho u^2 + P(\rho)\right) + \dfrac{2}{r}\rho u^2 = 0\,. \end{cases} \tag{1.4}$$

In this case global weak solutions are known to exist for the case $\gamma = 1$ (see [12], [13], [14], [15] and [16]) outside a solid ball at the origin. In [11], T. Makino et Takeno have obtained local weak solutions for the case $\gamma > 1$ But they can not obtain global weak solutions mainly due to the fact that they can not obtain uniform L^∞ estimates for the approximate solutions constructed by the Lax-Friedrichs scheme. For the one dimensional case, this problem is solved (see [2],[22]) for the approximate solutions with little viscosity. Recently, G. Q. Chen [1] had succeeded to overcome this difficulty by using another approximate solutions constructed by the Godunov scheme.

In this paper we shall show another approach for this problem (1.4). By using kinetic formulations for (1.4), we succeeded to obtain new estimates. This estimate is very interesting one because it holds for the case that the domain contains the origin.

2. Kinetic formulations of (1.4)

In this section we shall define the kinetic formulation of (1.4). Consider (1.4) in the domain $r \geq R_0$ with the initial boundary condition

$$u(0,r) = u_0(r)\ ,\ \rho(0,r) = \rho_0(r)\ ,\ (r \geq R_0) \tag{2.1}$$

$$u|_{r=R_0} = 0\ . \tag{2.2}$$

Here, we restrict ourselves to the case where the pressure p is given by

$$p = \kappa\rho^\gamma\ ,\ \ \kappa = \frac{\theta^2}{\gamma}\ ,\ \ \theta = \frac{\gamma-1}{2}\ , \tag{2.3}$$

where $\gamma > 1$ is a given constant.

Remark 1. From (1.4), we have

$$\left\{r^2\left(\frac{1}{2}\rho u^2 + \frac{1}{\gamma-1}p\right)\right\}_t + \left\{r^2\left(\frac{1}{2}\rho u^2 + \frac{\gamma}{\gamma-1}p\right)u\right\}_r = 0$$

So we shall say that $(\rho,\rho u)$ has a finite kinetic energy if it satisfies

$$E(\rho,\rho u) = \int_{R_0}^\infty r^2\left(\frac{1}{2}\rho u^2 + \frac{1}{\gamma-1}p\right)dr < \infty. \tag{2.4}$$

The homogeneous part of (1.4) is given by

$$\begin{cases} \dfrac{\partial}{\partial t}\rho + \dfrac{\partial}{\partial r}(\rho u) = 0\ , \\ \dfrac{\partial}{\partial t}(\rho u) + \dfrac{\partial}{\partial r}\Big(\rho u^2 + P(\rho)\Big) = 0\ . \end{cases} \tag{2.5}$$

$(\eta(\rho,\rho u), H(\rho,\rho u))$ is called an entropy pair if η is convex and satisfies

$$\frac{\partial}{\partial t}\eta(\rho,\rho u) + \frac{\partial}{\partial r}H(\rho,\rho u) = 0, \tag{2.6}$$

for smooth solutions of (2.5). To be more precisely, (η, H) satisfies

$$H_\rho = u\eta_\rho + \frac{p'(\rho)}{\rho}\eta_u\ ,\ \ H_u = \rho\eta_\rho + u\eta_u\ .$$

It is well-known that an entropy pair (η, H) for (2.5) is given by, for any convex function $g(\xi)$,

$$\begin{cases} \eta(\rho,\rho u) = \displaystyle\int_{-\infty}^\infty g(\xi)\chi(\rho;\xi-u)d\xi \\ H(\rho,\rho u) = \displaystyle\int_{-\infty}^\infty g(\xi)[\theta\xi + (1-\theta)u]\chi(\rho;\xi-u)d\xi\ , \\ \chi(\rho;\xi-u) = (\rho^{\gamma-1} - (\xi-u)^2)_+^\lambda\ , \\ \lambda = \dfrac{3-\gamma}{2\gamma-1}\ , \end{cases} \tag{2.7}$$

where $(x)_+ = max(x,0)$. Note that η is convex in $(\rho,\rho u)$-plane. For the detail, see [4] and [8].

Remark 2. Note that η is not convex in (ρ, u)-plane.

Remark 3. If we choose $g(\xi) = \xi^2/2$, then the entropy becomes

$$\eta_E = \frac{1}{2}\rho u^2 + \frac{1}{\gamma - 1}p,\ H_E = \left(\frac{1}{2}\rho u^2 + \frac{1}{\gamma-1}p\right) u\,.$$

In this case we obtain the energy. See Remark 1.

Now we are ready to give the kinetic formulation for (1.4). Put

$$U = \begin{pmatrix} \rho \\ \rho u \end{pmatrix},\quad F(U) = \begin{pmatrix} \rho u \\ \rho u^2 + p(\rho) \end{pmatrix},\quad G(U) = \begin{pmatrix} \dfrac{2\rho u}{r} \\ \dfrac{2\rho u^2}{r} \end{pmatrix},$$

Then (1.4) becomes

$$U_t + F(U)_r + G(U) = 0\,.$$

Multiplying both sides by $\nabla\eta(U)$ ($\nabla = (\frac{\partial}{\partial\rho}, \frac{\partial}{\partial(\rho u)})$) we get

$$\frac{\partial}{\partial t}\eta(\rho, \rho u) + \frac{\partial}{\partial r}H(\rho, \rho u) + \frac{2}{r}\rho u \eta_\rho(\rho, \rho u) + \frac{2}{r}\rho u^2 \eta_{\rho u}(\rho, \rho u) = 0$$

Definition 2.1. *$(\rho, \rho u)$ is an entropy solution of (1.4) if it satisfies*

$$\frac{\partial}{\partial t}\eta(\rho, \rho u) + \frac{\partial}{\partial r}H(\rho, \rho u) + \frac{2}{r}\rho u \eta_\rho(\rho, \rho u) + \frac{2}{r}\rho u^2 \eta_{\rho u}(\rho, \rho u) \le 0 \tag{2.8}$$

for all convex entropy pair (η, H) in the distribution sense.

Let us define the distribution $m(t, r, \xi)$ by

$$\frac{\partial}{\partial t}\chi + \frac{\partial}{\partial r}\{[\theta\xi + (1-\theta)u]\chi\} + \frac{2}{r}\rho u \chi_\rho + \frac{2}{r}\rho u^2 \chi_{\rho u} = -m_{\xi\xi}. \tag{2.9}$$

Then we derive, by (2.7),

$$\begin{aligned} &\int_{-\infty}^{\infty} g(\xi)\frac{\partial}{\partial t}\chi\, d\xi + \int_{-\infty}^{\infty} g(\xi)\frac{\partial}{\partial r}\{[\theta\xi + (1-\theta)u]\chi\}d\xi \\ &+ \int_{-\infty}^{\infty} g(\xi)\frac{2}{r}\rho u \chi_\rho d\xi + \int_{-\infty}^{\infty} g(\xi)\frac{2}{r}\rho u^2 \chi_{\rho u} d\xi = -\int_{-\infty}^{\infty} g(\xi) m_{\xi\xi}(t, r, \xi) \end{aligned} \tag{2.10}$$

Choosing again $g(\xi) = \frac{\xi^2}{2}$, we have

$$\begin{aligned} &\frac{\partial}{\partial t}\int_{-\infty}^{\infty} \frac{\xi^2}{2}\chi r^2 d\xi + \int_{-\infty}^{\infty} \frac{\xi^2}{2}\frac{\partial}{\partial r}\{[\theta\xi + (1-\theta)u]r^2\chi\}d\xi - \int_{-\infty}^{\infty} \frac{\xi^2}{2} 2r\{[\theta\xi + (1-\theta)u]\chi\}d\xi \\ &+ \int_{-\infty}^{\infty} \frac{\xi^2}{2}(2r\rho u \chi_\rho + 2r\rho u^2 \chi_{\rho u})d\xi = -\int_{-\infty}^{\infty} r^2 \frac{\xi^2}{2} m_{\xi\xi}(t, r, \xi) \end{aligned}$$

Thus we derive, using (η_E, H_E),

$$\begin{aligned} &\frac{\partial}{\partial t}r^2\eta_E + \frac{\partial}{\partial r}r^2 H_E - 2rH_E + 2r\rho u \frac{\partial}{\partial\rho}\eta_E + 2r\rho u^2 \frac{\partial}{\partial\rho u}\eta_E \\ &= \frac{\partial}{\partial t}r^2\eta_E + \frac{\partial}{\partial r}r^2 H_E = -\int_{-\infty}^{\infty} r^2 \frac{\xi^2}{2} m_{\xi\xi}(t, r, \xi) \end{aligned} \tag{2.11}$$

Integrating (2.11) over $[R_0, \infty) \times [0, T]$,

$$\int_{R_0}^{\infty} r^2\eta_E(T,r) - r^2\eta_E(0,r)dr = -\int_0^T \int_{R_0}^{\infty} \int_{-\infty}^{\infty} r^2 dm(t,r,\xi) \tag{2.12}$$

If $(\rho, \rho u)$ has a finite energy, the left-hand side of (2.12) is finite. Now the definition of entropy solutions become more simple way by the following theorem. Suppose that $(\rho, \rho u) \in L^\infty(R_+; L^1(R_0, \infty))$ is a weak solution with finite energy.

Theorem 2.2. *$(\rho, \rho u)$ is an entropy solution of (1.4) if and only if there exists a positive bounded measure m which satisfies*

$$\begin{cases} \dfrac{\partial}{\partial t}\chi + \dfrac{\partial}{\partial r}\{[\theta\xi + (1-\theta)u]\chi\} + \dfrac{2}{r}\rho u \chi_\rho + \dfrac{2}{r}\rho u^2 \chi_{\rho u} = -m_{\xi\xi} \\ \displaystyle\int_0^T \int_{R_0}^{\infty} \int_{-\infty}^{\infty} r^2 dm(t,r,\xi) < \infty \text{ ; } m \text{ : positive measure.} \end{cases} \tag{2.13}$$

Remark 4. If $(\rho, \rho u)$ is a classical solution in the domain Ω, $m(t,r,\xi) \equiv 0$ for $(t,r,\xi) \in \Omega \times (-\infty, \infty)$.

Remark 5. (2.12) means that in case $(\rho, \rho u)$ is not smooth, the energy $\int_{R_0}^{\infty} r^2\eta_E(t,r)dr$ is monotone decreasing function with respect to t. In other words, the entropy η_E is not conserved.

Proof. Suppose that $(\rho, \rho u)$ is an entropy solution. Let us define the distribution $m(t,r,\xi)$ by (2.9). Multiplying (2.9) by $g(\xi)$ and integrating over ξ, we get

$$\begin{aligned} &\frac{\partial}{\partial t}\eta + \frac{\partial}{\partial r}H + \frac{2}{r}\rho u \eta_\rho + \frac{2}{r}\rho u^2 \eta_{\rho u} \\ &= -\int_{-\infty}^{\infty} g(\xi) m_{\xi\xi}(t,r,\xi) = -\int_{-\infty}^{\infty} g''(\xi) dm(t,r,\xi) \end{aligned} \tag{2.14}$$

Since (2.8) holds for any convex function $g(\xi)$, $m(t,r,\xi)$ is a positive measure. If we consider the case $g(\xi) = \frac{\xi^2}{2}$, we derive the second equation of (2.13) by(2.12). The sufficiency of (2.13) can be proved in the same way. □

3. Estimates for entropy solution

For simplicity, we assume $\gamma = 3$ in this section. In this case (2.13) becomes very simple one. First, observe that

$$\chi = \left(\rho^{\gamma-1} - (\xi - u)^2\right)_+^{\lambda} = 1_{[u-\rho, u+\rho]}(\xi) \, .$$

Thus an entropy becomes, by (2.6)

$$\eta = \int_{u-\rho}^{u+\rho} g(\xi) d\xi \, .$$

Then we have

$$\begin{aligned}
&\frac{2}{r}\rho u \eta_\rho + \frac{2}{r}\rho u^2 \eta_{\rho u} \\
&= \frac{2}{r}\rho u \left(g(u+\rho) + g(u-\rho) - \frac{u}{\rho}g(u+\rho) + \frac{u}{\rho}g(u-\rho)\right) \\
&+ \frac{2}{r}\rho u^2 \left(\frac{1}{\rho}g(u+\rho) - \frac{1}{\rho}g(u-\rho)\right) \\
&= \frac{2}{r}\rho u \left(g(u+\rho) + g(u-\rho)\right).
\end{aligned}$$

Thus we have

$$\frac{2}{r}\rho u \chi_\rho + \frac{2}{r}\rho u^2 \chi_{\rho u} = \frac{2\rho u}{r}\left[\delta_{u-\rho}(\xi) + \delta_{u+\rho}(\xi)\right] \, .$$

Now (2.13) becomes

$$(3.1) \qquad \begin{cases} \dfrac{\partial}{\partial t}\chi + \xi\dfrac{\partial}{\partial r}\chi + \dfrac{2\rho u}{r}\left[\delta_{u-\rho}(\xi) + \delta_{u+\rho}(\xi)\right] = -m_{\xi\xi} \\ \displaystyle\int\int\int r^2 dm(t,r,\xi) < \infty \; ; \; m : \text{positive measure.} \end{cases}$$

Theorem 3.1. *Suppose that $(\rho, \rho u)$ is an entropy solution with finite energy. Then we have*

$$(3.2) \qquad \begin{cases} \displaystyle\sup_{r \geq R_0} \int_0^T (\rho^4 + \rho|u|^3) r^2 dt \leq CE_0 \\ \displaystyle E_0 = \int_{R_0}^{\infty} r^2 \eta_E(0,r) dr < \infty \, . \end{cases}$$

Proof. Since $(\rho, \rho u)$ has a finite energy, we have

$$(3.3) \qquad \int_{R_0}^{\infty}\int_0^{\infty} r^2\xi^2\chi(t,r,\xi) d\xi dr \leq 2E_0 \;\text{ for any } t \in [0,T] \, .$$

Multiplying (3.1) by $r^2 \times 1_{r \geq x} \times |\xi|\xi$ ($x \geq R_0$) and integrating over $[0,T] \times [R_0, \infty] \times (-\infty, \infty)$, we get

$$(3.4) \qquad \begin{aligned} &\int_{-\infty}^{\infty}\int_x^{\infty} r^2|\xi|\xi(\chi(T,r,\xi) - \chi(0,r,\xi)) dr d\xi + \int_0^T\int_{-\infty}^{\infty}\int_x^{\infty} r^2|\xi|\xi^2\frac{\partial}{\partial r}\chi \, dr d\xi dt \\ &+ \int_0^T\int_x^{\infty} 2\rho u r\left\{|u+\rho|(u+\rho) + |u-\rho|(u-\rho)\right\} dr dt = -\int_0^T\int_{-\infty}^{\infty}\int_x^{\infty} r^2|\xi|\xi \, m_{\xi\xi} \end{aligned}$$

Let us estimate the left-hand side of (3.4).

The first term of (3.4)

$$\left|\int_0^T \int_x^\infty r^2|\xi|\xi(\chi(T,r,\xi)-\chi(0,r,\xi))drd\xi\right|$$
$$\leq \int_{-\infty}^\infty \int_{R_0}^\infty r^2\xi^2\chi(T,r,\xi)drd\xi + \int_{-\infty}^\infty \int_{R_0}^\infty r^2\xi^2\chi(0,r,\xi)drd\xi \leq 4E_0 .$$

The second term of (3.4)+The third term of (3.4)

$$\int_0^T \int_{-\infty}^\infty \int_x^\infty r^2|\xi|\xi^2 \frac{\partial\chi}{\partial r} drd\xi dt + \int_0^T \int_x^\infty 2\rho ur\left\{|u+\rho|(u+\rho)+|u-\rho|(u-\rho)\right\} drdt$$
$$= \int_0^T \int_{-\infty}^\infty \left[r^2|\xi|\xi^2\chi\right]_x^\infty d\xi dt - \int_0^T \int_x^\infty \int_{-\infty}^\infty 2r|\xi|\xi^2\chi d\xi drdt$$
$$+ \int_0^T \int_x^\infty 2\rho ur\left\{|u+\rho|(u+\rho)+|u-\rho|(u-\rho)\right\} drdt$$
$$= -\int_0^T \int_{-\infty}^\infty x^2|\xi|\xi^2\chi(t,x,\xi)d\xi dt + \int_0^T \int_x^\infty 2\rho ur\left\{|u+\rho|(u+\rho)+|u-\rho|(u-\rho)\right\} drdt$$
$$- \int_0^T \int_x^\infty \int_{-\infty}^\infty 2r|\xi|\xi^2\chi d\xi drdt$$

Put

$$P = -\int_{-\infty}^\infty 2r|\xi|\xi^2\chi d\xi + 2\rho ur\left\{|u+\rho|(u+\rho)+|u-\rho|(u-\rho)\right\} .$$

Then we have

(i) $0 \leq u-\rho \leq u+\rho$

$$P = -2r\int_{u-\rho}^{u+\rho} \xi^3 d\xi + 2\rho ur\left\{(u+\rho)^2+(u-\rho)^2\right\}$$
$$= -2r \times \frac{1}{4} \times \left\{(u+\rho)^4-(u-\rho)^4\right\} + 2\rho ur(2u^2+2\rho^2) = 0 .$$

(ii) $u-\rho \leq 0 \leq u+\rho$

$$P = -2r\int_{u-\rho}^{0} -\xi^3 d\xi - 2r\int_0^{u+\rho} \xi^3 d\xi + 2\rho ur\left\{(u+\rho)^2-(u-\rho)^2\right\}$$
$$= -r(u^2-\rho^2)^2 \leq 0 .$$

(iii) $u-\rho \leq u+\rho \leq 0$

$$P = 2r\int_{u-\rho}^{u+\rho} \xi^3 d\xi - 2\rho ur\left\{(u+\rho)^2+(u-\rho)^2\right\} = 0 .$$

Thus we obtain

$$\int_0^T \int_{-\infty}^\infty x^2|\xi|\xi^2\chi(t,x,\xi)d\xi dt$$
$$(3.5) = \int_{-\infty}^\infty \int_x^\infty r^2\xi|\xi|(\chi(T,r,\xi)-\chi(0,r,\xi))drd\xi + \int_0^T \int_x^\infty P drd\xi + \int\int\int r^2|\xi|\xi dm_{\xi\xi}$$
$$\leq \int_{-\infty}^\infty \int_x^\infty r^2\xi|\xi|(\chi(T,r,\xi)-\chi(0,r,\xi))drd\xi + \int\int\int r^2|\xi|\xi dm_{\xi\xi}$$
$$\leq 3E_0 .$$

The following lemma can be proved very easily.

Lemma 3.2. *There exists a constant δ such that*

$$\int_{-\infty}^{\infty} |\xi|^3 \chi d\xi = \int_{u-\rho}^{u+\rho} |\xi|^3 d\xi \geq \delta\rho(|u|^3 + \rho^3), \tag{3.6}$$

Applying (3.6) to (3.5), we obtain (3.2). □

Remark 6. For the more general case ($\gamma \neq 3$), The following estimate also holds.

$$\sup_{r \geq R_0} \int_0^T \left(\rho|u|^3 + \rho^{\frac{3\gamma-1}{2}}\right) r^2 dt \leq CE_0 . \tag{3.7}$$

Remark 7. (3.2) and (3.7) also hold for the case $R_0 = 0$.

Remark 8. In general, the following Lemma holds. For the proof, see [8].

Lemma 3.3. *There exists a constant δ depending on γ such that*

$$u \int_{-\infty}^{\infty} |\xi|\xi\chi d\xi \geq \delta\rho|u|^2(\rho^\theta + |u|), \tag{3.8}$$

$$\int_{-\infty}^{\infty} |\xi|^3 \chi d\xi \geq \delta\rho(|u|^3 + \rho^{3\theta}), \tag{3.9}$$

$$\int_{-\infty}^{\infty} \xi(\xi - u)|\xi|\chi d\xi \geq \delta\rho(\rho^{3\theta} + \rho^{\gamma-1}|u|) . \tag{3.10}$$

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