Weak solutions with a shock to a model system of the radiating gas

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Abstract

The system of equations for the polytropic gas with the flow of radiative heat flow is approximated by the system of a hyperbolic conservation law and a linear elliptic equation, which is called a model system of the radiating gas. We discuss about these approximation and related results of the system thus obtained.

At first we notice that for the model system of the radiating gas, the spatial derivative of the initial data is smaller than a certain negative critical value, the solution blows up. Thus it is necessary and natural to think about weak solutions in a suitable sense.

Mainly we talk about the Cauchy problem for the model system of a radiating gas with Riemann initial data since this initial data give rise to a discontinuity of the solution. The assumption that the left state u_- is larger than the right state u_+ ensures the existence of a corresponding traveling wave, connecting the left state u_- and the right state u_+ asymptotically. Although the solution has a discontinuity, the uniqueness of a solution in a weak sense is established by imposing the entropy condition. Furthermore, if the magnitude of discontinuity is smaller than or equals to $\frac{1}{2}$, the global solution exists and tends to the traveling wave as time goes to infinity.

1 Introduction

The system of equations for the polytropic gas with radiative heat flow is introduced in [14]:

$$\begin{cases} \rho_t + (\rho u)_x = 0, \\ (\rho u)_t + (\rho u^2 + p)_x = 0, \\ \{\rho(\frac{1}{2}u^2 + e)\}_t + \{\rho u(\frac{1}{2}u^2 + e) + up + q\}_x = 0, \\ p = \rho R\theta = A\rho^\gamma \exp \frac{(\gamma-1)s}{R}, \\ -q_{xx} + 3\alpha^2 q + 4\alpha\sigma(\theta^4)_x = 0 \end{cases} \tag{1}$$

where ρ is density, u velocity, p the pressure, e the internal energy, q the radiative heat-flux, R the gas constant, s is the entropy, $\theta > 0$ the absolute temperature, $\gamma > 1$ the (constant) rate of specific heats and A a positive constant. σ is the Stefan-Boltzmann constant.
We consider the pressure p, absolute temperature θ and internal energy e as functions of the density ρ and the entropy s:

$$p = p(\rho, s), \quad \theta = \theta(\rho, s) \quad \text{and} \quad e = e(\rho, s). \tag{2}$$

The thermodynamic law is expressed as

$$e_\rho = \frac{p}{\rho^2}, \quad \text{and} \quad e_s = \theta. \tag{3}$$

By standard computation using (3), we may rewrite (1) as:

$$\begin{cases} \rho_t + (\rho u)_x = 0, \\ \rho(u_t + uu_x) + p_x = 0, \\ \rho\theta(s_t + us_x) + q_x = 0, \\ p = \rho R\theta = A\rho^\gamma \exp \frac{(\gamma-1)s}{R}. \\ -q_{xx} + 3\alpha^2 q + 4\alpha\sigma(\theta^4)_x = 0 \end{cases} \tag{4}$$

We assume the Stefan-Boltzmann constant σ is small and expressed as $\sigma = \varepsilon\sigma_0$, where ε is a dimensionless small parameter and σ_0 is a positive constant.

As an equilibrium state of the gas is considered to be the state in which any flow does not occur, we define it as the state satisfying

$$(\rho, u, s, q) = (\rho_0, 0, s_0, 0),$$

where ρ_0 and s_0 are positive constants. Following the idea of [3], we expand a state (ρ, u, s, q) around the equilibrium state $(\rho_0, 0, s_0, 0)$ as

$$\begin{cases} \rho = \rho_0 + \varepsilon\bar{\rho}(\bar{x}, \bar{t}), \\ u = \varepsilon\bar{u}(\bar{x}, \bar{t}), \\ s = s_0 + \varepsilon^2\bar{s}(\bar{x}, \bar{t}), \\ q = \varepsilon^2\bar{q}(\bar{x}, \bar{t}) \end{cases} \tag{5}$$

where $\bar{\rho}$, $\bar{u}$, $\bar{s}$ and $\bar{q}$ are functions of $\bar{t} = \varepsilon t$ and $\bar{x} = x - C_0 t$. Here, C_0 is the sound speed, given by

$$C_0 = \sqrt{\frac{\partial p}{\partial \rho}(\rho_0, s_0)} = \sqrt{\gamma\frac{p_0}{\rho_0}} = \sqrt{\gamma R\theta_0}. \tag{6}$$

Expanding p and θ around the equilibrium state $(\rho_0, 0, s_0, 0)$, we have that

$$p = p_0 + \varepsilon C_0^2\bar{\rho} + \varepsilon^2(\gamma - 1)(\frac{C_0^2}{2\rho_0}\bar{\rho}^2 + \rho_0\theta_0\bar{s}) + O(\varepsilon^3) \tag{7}$$

and

$$\theta = \theta_0 + \varepsilon(\gamma - 1)\frac{\theta_0}{\rho_0}\bar{\rho} + O(\varepsilon^2). \tag{8}$$

Substituting these expansion in the system (4), retaining $O(\varepsilon^2)$ with neglecting $O(\varepsilon^3)$ and making some modifications, we obtain the following simplified system of equations, which we call the model system of the radiating gas:

$$\begin{cases} u_t + uu_x + q_x = 0, \\ \\ -q_{xx} + q + u_x = 0. \end{cases} \tag{9}$$

The first equation of (9) is a conservation law and the second is a elliptic equation. As the second equation of (9) is a linear elliptic equation, we may express q in terms of u formally;

$$q = -Ku_x \tag{10}$$

where

$$(Kf)(x) = (-\frac{\partial^2}{\partial x^2} + 1)^{-1} = \frac{1}{2}\int_{-\infty}^{\infty} e^{-|x-y|}f(y)dy. \tag{11}$$

By (10), we have that

$$q_x = -Ku_{xx} = u - Ku. \tag{12}$$

Here we would like to note that approximation until $O(\varepsilon)$ gives viscous Burgers' equation

$$u_t + uu_x = u_{xx}. \tag{13}$$

Approximation until $O(1)$ gives inviscid Burgers' equation

$$u_t + uu_x = 0. \tag{14}$$

2 Survey of Relating Results

Although our main concern is the weak solutions of (9), we would like to survey the relating results about (9) as well as (1) here. The system of equations (9) is derived as the third order approximation from the motion of radiating gas with thermo-non-equilibrium equation (1), as we have seen. We should refer [14], [1] and [2] for the physical description about (1). Hamer [3] studied (9) with the interest from a physical point of view. Especially he has studied the steady progressive shock-wave solutions.

Mathematically, S. Kawashima and Y. Tanaka have started to research (9) with [11]. They proved the local existence and uniqueness of the smooth solution under suitable conditions. Furthermore, the asymptotic behavior is obtained for the cases, $u_- = u_+$ and $u_- < u_+$. The first case gives diffusion waves and the second one rarefaction waves. In one word, these results implies that the asymptotic behavior of the solutions of (9) is well- approximated by (13) for these spatial asymptotic conditions $u_- \leq u_+$. Also, K. Ito [6] proved the uniqueness of the weak solution and gave the convergent rate toward rarefaction waves, assuming that $u_- \leq u_+$.

Kawashima and Nishibata have researched (9) with the spatial asymptotic condition $u_- > u_+$ in [8]. They have shown the existence of smooth or discontinuous traveling waves, and also proved the uniqueness of these traveling waves in the class of piecewise smooth functions, under the entropy condition. In brief, the condition $u_- > u_+$ is necessary and sufficient to ensure the existence of traveling waves in a weak sense. The magnitude of the quantity $|u_- - u_+|$ is shown to give information on the smoothness of the traveling waves. The smaller $|u_- - u_+|$ gives the more times differentiability. Furthermore, they have shown that C^3-smooth traveling waves are asymptotically stable and that the rate of convergence toward these waves is $t^{-\frac{1}{4}}$.

In [9], Kawashima and Nishibata have studied weak solutions to (9). They show that if the spatial derivative of the initial data is larger than a certain negative critical value, a unique solution exists globally in time. But if it is smaller than another negative critical value, then the spatial derivative of the corresponding solution blows up in a finite time. Then they start to analyze solutions in a weak sense. The system of equations (9) with the Riemann initial data has a admissible global solution, which tends to the corresponding traveling waves. These results are introduced in the third and second chapters of the present paper, briefly. Please refer to [9] for details.

Kawashima, Nishibata and Nikkuni ([10]) have treated much more general systems of hyperbolic conservation laws coupled with elliptic equations than (9) and (1). After showing the equivalence between the

symmetrizability of the system and the existence of the strictly convex entropy, they analyze the symmetrized system. They show the global existence of solutions for a small initial data in a suitable Sobolev space, with assuming the stability condition. The global existence is proved by a standard energy method. Furthermore, it is shown that the solution decays to zero in the Sobolev space. Moreover, if the system is strictly hyperbolic, the solution is well approximated by diffusion waves for a large time. Actually, by the spectral analysis they show that a solution

approaches to a solution of the corresponding hyperbolic conservation laws with viscous terms, with the rate $t^{-\frac{3}{4}+c}$ (c is a arbitrarily small constant). Thus their theorem follows from the well-known fact that the solution of viscous hyperbolic conservation laws approaches to the diffusion waves, with the rate $t^{-\frac{1}{2}+c}$.

The Riemann problem is extensively researched by many authors, especially to the inviscid conservation laws. On the contrary this problem for conservation laws with some source terms is ignored because of the difficulty which come from discontinuities of solutions, nonetheless its physical importance. As far as we know, Hsiao and Greenberg [4] considered the Riemann problem for the conservation laws with relaxation terms. T. P. Liu and Hoff [5] researched this problem of the 2×2 navier-stokes equations.

3 Preliminary results

We study the system (9) with the initial condition:

$$u(x,0) = u_R(x) = \begin{cases} u_- & \text{for } x < 0 \\ u_+ & \text{for } x > 0. \end{cases} \tag{15}$$

It is shown in [9] that when the spatial derivative of the initial data, $u_{0,x}(x_0)$, is larger than a critical value at certain x_0, the spatial derivative of the solution of (9) blows up in the finite time. Thus it is natural to think about weak solutions in a suitable sense.

We define weak solutions, as follows (see, [6], [8] and [12]).

Definition 3.1 *We define admissible solutions $(u,q)(x,t)$ in weak sense as functions*

$$u \in L^\infty(\mathbf{R}\times(0,T)) \qquad \text{and} \qquad q \in L^\infty(\mathbf{R}\times[0,T]),$$

which satisfy

$$\int_0^T\int_{-\infty}^{+\infty} |u-k|\varphi_t + \operatorname{sign}(u-k)(\frac{1}{2}u^2 - \frac{1}{2}k^2)\varphi_x - \operatorname{sign}(u-k)(u-Ku)\varphi dx\, dt \geq 0 \tag{16}$$

and

$$\int_{-\infty}^{+\infty} -q\psi_{xx} + q\psi - u\psi_x dx = 0 \tag{17}$$

where φ is any non negative function in $C_0^\infty(\mathbf{R}\times(0,T))$, ψ is arbitrary rapidly decreasing function over $\mathbf{R}$ and k is any number in $\mathbf{R}$. Furthermore, the function sign *is defined as*

$$\operatorname{sign}(x) = \begin{cases} -1 & \text{for } x<0 \\ 0 & \text{for } x=0 \\ 1 & \text{for } x>0. \end{cases} \tag{18}$$

As prototype of the discontinuous solutions, our concerns go to the Riemann initial data:

$$u_0(x) = \begin{cases} u_- & x<0 \\ u_+ & x>0. \end{cases} \tag{19}$$

The uniqueness of the admissible solution is shown in the space of bounded variation factions $u(x)$ satisfying

$$(u-u_R)(x) \in L^1 \cap L^\infty. \tag{20}$$

This result is proved by the similar computation as the corresponding theorem in [6].

If the solution (u,q) is a piecewise smooth function in L^∞, it is easy to show the equivalence between the above definition and that "(u,q) satisfies the the Rankine-Hugoniot condition

$$x'(t) = \frac{1}{2}(u_r+u_l),$$
$$[q]=0, \tag{21}$$
$$[u-q_x]=0$$

and entropy condition

$$u_r < u_l \tag{22}$$

at $x = x(t)$", and "(u,q) satisfies (9) except for $x = x(t)$, where the curve $x = x(t)$ is the set of the points on which discontinuities appear". This equivalence is proved by the same discussion as the corresponding theorem in [8]. Here, we express left and right limits at discontinuities by f_l and f_r. Also, $[f] := f_r - f_l$. From the Rankine-Hugoniot condition (21), we find out that the magnitude of discontinuities should decay exponentially fast, provided that $u_x > -1$.

Lemma 3.2 *Magnitude of discontinuities $|u_l - u_r|$ decays exponentially if the inequality, $\frac{1}{2}(u_{x,r} + u_{x,l}) \geq c > -1$, holds for arbitrary $t > 0$.*

The above lemma is proved by direct calculation, using (9) and (21). The traveling waves are solutions of (9), which is expressed in the form $(u,q)(x,t) = (U,Q)(\xi) \quad (\xi = x - st)$, where s is the constant called the speed of traveling waves. We will see that the initial value problem (9) and (15) has a global admissible solution and its asymptotic state is the traveling wave $(U,Q)(\xi)$ which connects u_- and u_+;

$$\lim_{\xi \to \pm\infty} U(\xi) = u_\pm. \tag{23}$$

The existence of the traveling wave solution is shown in [8].

Theorem 3.3 *(1) If $|u_+ - u_-| \leq \sqrt{2}$ and $u_- > u_+$, then a traveling wave $(U,Q)(x - st)$, satisfying (23), exists uniquely up to shift. Choosing shift parameter suitably, traveling wave (U,Q) has symmetric property.*

$$(U,Q)(-\xi) = (-U,Q)(-\xi) \qquad \textit{where} \qquad \xi = x - st. \tag{24}$$

Furthermore, the traveling wave speed s equals to $\frac{u_+ + u_-}{2}$ and $U'(\xi) < 0$.
Moreover, $U(\xi)$ is in B^1 and $Q(\xi)$ is in B^2 hold.
(2) If $|u_+ - u_-| < \frac{2\sqrt{2n}}{n+1}$, then $U(\xi)$ is in B^n and $Q(\xi)$ is in B^{n+1}, where $n = 2, 3, \cdots$.
(3) If $|u_- - u_+| \leq \sqrt{2}$, then

$$|U| \leq \frac{1}{2}|u_- - u_+| \quad \textit{and} \quad |U'| \leq \frac{1}{4}|u_- - u_+|^2. \tag{25}$$

Moreover, if $|u_- - u_+| < \frac{\sqrt{6}}{2}$,

$$|U''| \leq \frac{|u_- - u_+|^3}{8(1 - 3|v_0|)} \tag{26}$$

where v_0 is the constant given by

$$v_0 = \frac{-1 + \sqrt{1 - \frac{1}{2}|u_- - u_+|^2}}{2}.$$

Here and hereafter we assume that the traveling wave satisfies (24). So, if $|u_- - u_+| \leq \sqrt{2}$, we have that

$$U(0) = 0. \tag{27}$$

In (9) and (15), changing independent variables as $(x,t) \to (x - \frac{u_+ + u_-}{2}t, t)$ and then changing unknown functions as $(u,q) \to (u - \frac{u_+ + u_-}{2}, q)$ we have the system (9) with the initial condition:

$$u_\pm = \mp\alpha, \tag{28}$$

where $\alpha = \frac{1}{2}|u_- - u_+|$. We impose this condition here and hereafter. We can show the existence of a local solution to the system (9) with initial constant data α for $x < 0$ and $-\alpha$ for $x > 0$ respectively. Let us note that the condition $\alpha > 0$ implies characteristics enter the line $\{x = 0\}$ from both sides. Thus it is apparent that this problem is well-posed although we do not pose any boundary conditions. The solution with initial data (28) has a following symmetric property.

Lemma 3.4

$$(u,q)(x,t) = (-u,q)(-x,t)$$

Therefore, once we obtain the smooth solution in the first quarter plane $\{0 < x < \infty, 0 \leq t\}$, we can extend it to a weak solution in $\{-\infty < x < \infty, 0 \leq t\}$. Because the above lemma and the local existence of smooth solution in $\mathbf{R}_+$ implies that discontinuities appear only on the line $\{x = 0\}$.

Thus, it is enough to consider the problems in the first quarter plane $\{x > 0, t \geq 0\}$. Now the problem is reformed to

$$\begin{cases} u_t + uu_x + q_x = 0, \\ -q_{xx} + q + u_x = 0. \end{cases} \quad \text{for} \quad x > 0, \quad t \geq 0 \tag{9$'$}$$

with negative initial condition:

$$u(x,0) = u_0(x) \leq c < 0 \quad \text{for} \quad x > 0. \tag{19$'$}$$

Local existence of this problem is given by the following theorem:

Lemma 3.5 *Suppose that $u_0 \in B^1(\mathbf{R}_+)$ such that $u_0 = u_0(x) \leq c < 0$ for $x > 0$, there is a solution in $B^1(\mathbf{R}_+ \times [0,T_0])$ satisfying*

$$u(x,t) < 0, \tag{29}$$

where T_0 is depending only on $|u_0'|_\infty$.
Moreover, if $u_0 - \alpha \in L^1 \cap H^2$ then $u(x,t) - \alpha \in L^1 \cap H^2$

Maximal principle is verified for the initial value problem in the first quarter plane.

Lemma 3.6 *Let $u(x,t) \in B^1(\mathbf{R}+\times[0,T])$ be the solution of the problem $(9)'$ and $(1.2)'$ with $u_0 \in B^1(\mathbf{R})$, $u_0 < 0$ and $u_0^{'} \leq 0$. Then $u(x,t)$ and $u_x(x,t)$ satisfies*

$$\hat{a}_0 \leq u(x,t) < 0, \tag{30}$$

$$u_x(x,t) \leq 0, \tag{31}$$

and

$$\hat{a}_0 \leq \{(Ku)(x,t)\}_x \tag{32}$$

for $(x,t) \in \mathbf{R}_+ \times [0,T]$.

The constant $\hat{a}_0$ in the above lemma are defined as:

$$\hat{a}_0 = \inf_{x>0} u_0(x).$$

The global existence and the lower bound for u_x is also given as follows. When $-\frac{1}{4} \leq \hat{a}_0 < 0$, we write that

$$\hat{v}_* = \frac{-1-\sqrt{1-4|a_0|}}{2} \quad \text{and} \quad \hat{v}^* = \frac{-1+\sqrt{1-4|a_0|}}{2}.$$

Theorem 3.7 *Suppose that $u_0 \in B^1(\mathbf{R}_+)$ and satisfies*

$$\hat{a}_0 \leq u_0(x) < 0 \tag{33}$$

and

$$\hat{v}^* \leq u_0^{'}(x) \leq 0 \quad \textit{for} \quad x \in \mathbf{R}_+, \tag{34}$$

for $-\frac{1}{4} \leq \hat{a}_0 < 0$, then (9)' and (15)' admits a global solution, which satisfies (30) and

$$\hat{v}^* \leq u_x(x,t) \leq 0 \tag{35}$$

for arbitrarily t.

It is easily seen that all of the assumptions in the above two theorems are satisfied in the Riemann problem (9) and (15). Thus we know that this problem has a global solution. Also, note that the weak solution, constructed in above theorem and extended to whole upper plane by using Lemma 3.4, satisfies the Rankine-Hugoniot condition (21) and the entropy condition (22).

Theorem 3.8 *If $|u_- - u_+| \leq \frac{1}{2}$, the initial value problem (9) and (15) has a unique admissible solution globally in time.*

Although the above theorem ensures the global existence of the solution with the Riemann initial data, it is still necessary to obtain the uniform estimate of $(u-U, q-Q)$ to get the asymptotic states. This is done through the energy estimate in the next section.

4 The main result

Throughout this section, $|\cdot|_n$ and $\|\cdot\|_n$ implies L^n-norm and H^n-norm over $\mathbf{R}_+ := \{x > 0\}$, respectively.

At first, we reform equations for simplicity. We express the perturbation from the traveling waves by ϕ and ψ:

$$\phi(x,t) = u(x,t) - U(x) \tag{36}$$

and

$$\psi(x,t) = q(x,t) - Q(x). \tag{37}$$

As (u, q) and (U, Q) are solutions of (9), (ϕ, ψ) satisfies

$$\phi_t + (U\phi + \frac{1}{2}\phi^2)_x + \psi_x = 0 \tag{38}$$

$$-\psi_{xx} + \psi + \phi_x = 0, \tag{39}$$

in the first quarter plane and the second quarter plane, respectively. From initial condition (15) and (28), ϕ satisfy

$$\phi_0(x) = \phi(x,0) = \begin{cases} \alpha - U(x) & \text{for } x < 0 \\ -\alpha - U(x) & \text{for } x > 0 \end{cases}. \tag{40}$$

Owing to Lemma 3.4, it is enough for us to consider the initial value problem in the first quarter plane $\{x > 0, t > 0\}$. We may define the anti-derivative of ϕ (see (46));

$$\Phi(x,t) = -\int_x^\infty \phi(y,t)dy \quad \text{for} \quad x > 0. \tag{41}$$

Initially Φ satisfies

$$\Phi_0(x) := \Phi(x,0) = -\int_x^\infty -\alpha - U(x)dx \quad \text{for} \quad x > 0. \tag{42}$$

The above indefinite integral converges since the function $U(x)$ decays to $-\alpha$ exponentially as $x \to \infty$. Thus (Φ, ψ) satisfies the equations derived by integrating (38) on the interval $(0, \infty)$,

$$\Phi_t + U\Phi_x + \frac{1}{2}\Phi_x^2 + \psi = 0. \tag{43}$$

Lemma 4.1

$$\phi(x,t) \leq 0 \quad for \quad x > 0, \tag{44}$$

$$\Phi(x,t) \geq 0 \quad for \quad x > 0. \tag{45}$$

The validity of the definition (41) follows from the lemma:

Lemma 4.2

$$|\phi(t)|_1 \leq |\phi_0|_1. \tag{46}$$

Applying energy calculation on (38) and (43) and then estimating the boundary integrals along $\{x=0\}$ by (44) and (45), we have the following H^2-estimate.

Lemma 4.3 *If $\alpha \leq \frac{1}{4}$, we have the uniform estimate*

$$\|(\Phi,\psi)(t)\|_2^2 + \int_0^t \|\phi(\tau)\|_1^2 + \|\psi(\tau)\|_2^2 d\tau + \int_0^t \int_0^\infty |U_x|\Phi^2 dx\, d\tau \leq C\|\Phi_0\|_2^2 \tag{47}$$

for arbitrary $t > 0$.

We are now at a position to state the one of main theorems in the present paper.

Theorem 4.4 *The initial value problem (9), (15) has a unique admissible solution $(u,q)(x,t)$ which satisfies (20), in sense of Definition 3.1 globally, such that*

$$\sup_x |(u(x,t) - U(x-st), q(x,t) - Q(x-st)| = O(t^{-\frac{1}{4}})$$

where (U,Q) is the traveling wave with (9), (23).

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