Mathematical Modeling and Simulation for Origami Tsunami Pod

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Abstract. Tsunami is caused by a large coastal earthquake. Spherical tsunami shelters were developed in order to avoid serious damages by tsunami. In this paper, we develop the three dimensional model named tsunami pod as a triangulated ellipsoid which can be folded flatly, conduct the numerical simulation by FEM software, and confirm the safety by the von Mises Equivalent value and Head Injury Criterion (HIC). As the result of the first simulation, it is proven that the initial model is weak to the impact force from rigid wall and obstacles and can be strained in the central part. On the other hand, an occupant hits against the internal wall and can be injured. So, the seat and upper and lower bars are added to the initial model. The von Mises equivalent value for the modified model is decreased, however is still over the one of the first simulation. While, HIC value is decreased largely which is much less than the HIC safety level. This is because an occupant does not hit on the internal wall for the occupant restraint system. Moreover, we optimize the form and property for the tsunami pod by minimizing its weight. Finally, we confirm the validity of the optimal model in the original condition of fluid-structure coupling analysis.

1. Introduction

In the Tohoku earthquake which occurred on March 11 of 2011, serious damages were caused by the tsunami after its earthquake. According to the survey of the Metropolitan Police Department of March 10 of 2015, 15,893 dead people, 6,152 injured people, and 2,572 missing people were confirmed. Since the earthquake, how to protect people’s lives from tsunami has been a big issue in Japan. The height of tsunami on land is the same as the several levels of buildings, and its speed is fast as an adult runs with full effort. It is assumed that the broken evacuation route would prevent people from moving, so elder people, disabled people, seriously injured people, and visitors would not easily evacuate to some higher places. As a result of that, they would be drowned and injured severely by obstacles. Recently, as a countermeasure for minimizing the damage occurred by a tsunami, an evacuation building and a floating type shelter have been proposed. These facilities are limited to an urgent and temporary use out of sheer necessity. The spherical shaped shelter which has evacuation space in an upper part and weight and saving space in a lower part was proposed by Shigematsu [7, 8]. And, its motion characteristics was examined under hydraulic experiment applying
a reduced model. However, neither experiment nor simulation for examining the motion of an occupant has never been conducted by them. For upgrading to the level of necessities from temporary evacuee facilities, it is essential to consider the performance of storage and conveyance with ensuring safety. As possible solution for storage, foldable structure is considered and an ellipsoid body is applied as its form. This structure is named as “tsunami pod”, because the form looks like a shell wrapping bean. For practical applications of this tsunami pod, it is necessary to examine the mechanism to be restrained after being the deployed situation from the folded situation similar to a space antenna, however we will leave the problem as future work.

In this paper, under the condition of coupling phenomenon between structure and fluid, we optimize the design to prevent a tsunami pod from being broken in hitting against rigid walls and obstacles while reducing its weight, and also examine modeling and simulation for design specification to prevent an occupant head from being injured by hitting against the internal side of a tsunami pod.

### Table 1. Commercially available tsunami shelters.

<table>
<thead>
<tr>
<th>Product name</th>
<th>Life Amour</th>
<th>HIKARI</th>
<th>Note</th>
<th>Super Barrier S4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Form on sale</td>
<td>Integrated</td>
<td>Integrated or Assembly</td>
<td>Integrated</td>
<td>Integrated</td>
</tr>
<tr>
<td>Material</td>
<td>CFRP</td>
<td>CFRP</td>
<td>CFRP</td>
<td>Steel</td>
</tr>
<tr>
<td>Form in use</td>
<td>Spherical body</td>
<td>Spherical body</td>
<td>Spherical body</td>
<td>Driacontahedron</td>
</tr>
<tr>
<td>Diameter</td>
<td>120 cm</td>
<td>120 cm</td>
<td>120 cm</td>
<td>180 cm</td>
</tr>
<tr>
<td>Weight</td>
<td>80 kg</td>
<td>80 kg</td>
<td>80 kg</td>
<td>500 kg</td>
</tr>
<tr>
<td>Maximum number of occupants</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>Withstand load</td>
<td>9 t</td>
<td>22.4 t</td>
<td>12.5 t</td>
<td>2,600 kg</td>
</tr>
<tr>
<td>Price</td>
<td>400,000 yen</td>
<td>500,000 yen</td>
<td>670,000 yen</td>
<td>2,600,000 yen</td>
</tr>
</tbody>
</table>

2. Mathematical modeling for commercially available tsunami pod

2.1. Consideration of commercially available tsunami shelters

Since the Tohoku earthquake, several kinds of tsunami shelters made of carbon fiber reinforced plastic (CFRP) have been produced by Japanese companies. Table 1 shows those specifications for 4 kinds of tsunami shelters. Concerning form, 3 of them are sphere and one of them is semi-regular dotriacontahedron. The spherical form is assumed to avoid the crash by obstacles and pressure from outside for its curved surface and to be able to diffuse the stress. As a result, it can decrease the load at each point, and suppress the explosion and damage to the utmost limit. CFRP applied into each product as material is a kind of plastic into which epoxy resin is poured to enhance strength not to be bent and crushed. It has superior
characteristics that it is 10 times stronger than iron and one-fifth in weight as the one. It has a good heat retaining property to prevent an occupant from heat and cold, and also resistant to corrosion deterioration. For each tsunami shelter, safety is examined from several other viewpoints such as air tightness and oxygen concentration. Furthermore, some of companies conducted a water fall test and floating test in the sea. Concerning a form, the product "HIKARi" in Table 1 is an assembling type which is carried in with each part split apart. After being carried into home, they have to put parts together into spherical form and keep it in balcony in its development state. On the other hand, as a tsunami pod which can be kept in foldable state and developed instantaneously in case of emergency, we develop a triangulated ellipsoid body which can be folded by pushing in one direction. This folding way for a sphere has already been researched in [5, 11]. Here, we should elucidate the mechanism which makes it possible to deploy a developed ellipsoid body in case of emergency and fold for being stored after use. However, in this paper as the first report, we assume that there is no influence on strength by being deployed and folded. We set its size as 1200 mm of radius for major axis and 800 mm of radius for minor axis and examine the case where one occupant boards a shelter.

2.2. Mathematical modeling for origami pod

The model of tsunami pod in the previous paper [4] is generated by calculating coordinates of an ellipsoid approximately, and it cannot be folded flatly. In this paper, as a new model of tsunami pod, we develop a triangulated ellipsoid which can be folded flatly. This triangulated ellipsoid does not have strain in the deployed state and the folded state. However, there is strain in the middle of deformation. This structure is often called the "bi-stability" in [2].

The following steps give a design method of a triangulated ellipsoid. Let \((x_0, y_0, z_0)\) be a vertex of the upper-half part of this ellipsoid \((z \geq 0)\). In this design method, we define a vertex of the lower-half part \((z \leq 0)\) as \((x_0, -y_0, -z_0)\). Thus we may consider the upper-half part of this ellipsoid. In the following (iv), the lower-half part is determined by the position of vertices of the upper-half part. Also we adopt the reversed spiral structure.

(i) Consider an ellipsoid written by the equation \(\frac{x^2+y^2}{a^2} + \frac{z^2}{b^2} = 1\), where \(a, b > 0\). As a representation by the polar coordinates, we rewrite the above equation as \(x = a \cos \theta \cos \varphi, \ y = a \sin \theta \cos \varphi, \ z = b \sin \varphi\), where \(\theta\) and \(\varphi\) are the latitude and the longitude, respectively.

(ii) Separate this ellipsoid by \(m\) planes which is parallel for the \(xy\) plane. Let \(P_k\) \((k = 0, 1, 2, \ldots, m - 1)\) be the boundary of a cross-section of this ellipsoid. Here we choose the boundary written as follows.

\[ P_k = \{ (x, y, z) \mid x^2 + y^2 = r_k^2, \ z = h_k \}, \quad (1) \]
where
\[ r_k = \frac{ab}{\sqrt{b^2 + a^2 \tan^2 \left( \frac{k\pi}{2m} \right)}}, \quad h_k = \frac{ab \tan \left( \frac{k\pi}{2m} \right)}{\sqrt{b^2 + a^2 \tan^2 \left( \frac{k\pi}{2m} \right)}}. \tag{2} \]

Figure 1 shows a graphical image of cross-sections for the side view.

![Graphical image of cross-sections](image)

Figure 1. A graphical image of cross-sections for the side view. The parameters are \(a = 800\), \(b = 1200\), \(m = 6\).

(iii) Replace the boundaries of the cross-sections \(P_k\) with similar \(n\)-sided regular polygons, where \(n\) is a natural number which is greater than or equal to 3. Suppose that each vertex is written by
\[ U_{j,k} = (x_{j,k}, y_{j,k}, z_{j,k}) \quad (j = 0, 1, 2, \cdots, n-1), \tag{3} \]

\[ x_{j,k} = r_k \cos \left( \frac{2\pi j}{n} + \sum_{i=0}^{k} \theta_i \right), \quad y_{j,k} = r_k \sin \left( \frac{2\pi j}{n} + \sum_{i=0}^{k} \theta_i \right), \quad z_{j,k} = h_k, \]

where we suppose \(\theta_0 = 0\) for simplicity, and \(\theta_k \in \mathbb{R} \quad (k = 1, 2, \cdots, m-1)\). If the inequality
\[ 0 < h_k - h_{k-1} < 2 \sqrt{r_k r_{k-1} \cos \left( \frac{\pi}{n} \right)} \tag{4} \]
holds, then \(\theta_k\) is determined explicitly. In this design method, we define the following formula of \(\theta_k\):
\[ \theta_k := (-1)^k \arccos \left( \frac{(h_k - h_{k-1})^2}{4 r_k r_{k-1}} + (-1)^{k+1} \sqrt{1 - \frac{(h_k - h_{k-1})^4}{16 r_k^2 r_{k-1}^2 \cos^2 \left( \frac{\pi}{n} \right)}} \sin \left( \frac{\pi}{n} \right) \right). \tag{5} \]

Thus triangular faces of the upper-half part of this ellipsoid are given by
\[ \triangle U_{j,k-1} U_{j+1,k-1} U_{j,k} \quad \text{and} \quad \triangle U_{j+1,k-1} U_{j,k} U_{j+1,k} \tag{6} \]
for $j = 0, 1, 2, \cdots, n - 1$ and $k = 1, 2, \cdots, m - 1$.

(iv) Each vertex of the lower-half part is determined by

$$L_{j,k} = (x_{j,k}, -y_{j,k}, -z_{j,k}).$$

Thus we obtain a triangulated ellipsoid of the lower-half part separated by triangular faces

$$\triangle L_{j,k-1}L_{j+1,k-1}L_{j,k} \text{ and } \triangle L_{j+1,k-1}L_{j,k}L_{j+1,k}$$

for $j = 0, 1, 2, \cdots, n - 1$ and $k = 1, 2, \cdots, m - 1$.

Throughout the above steps, we can obtain a triangulated ellipsoid, where the top face and the bottom face are a same $n$-sided regular polygon. Figure 2 shows an example of a triangulated ellipsoid determined by the parameters $a = 800$, $b = 1200$, $n = 10$, $m = 6$. Figure 3 shows origami parts for Figure 2 and its pictures. Each origami part is obtained by applying the design method of [12].

![Figure 2. A triangulated ellipsoid. The parameters are $a = 800$, $b = 1200$, $n = 10$, $m = 6$.]

### 2.3. Mathematical modeling for pod and occupant

Since the coordinates of the tsunami pod have already determined for the parameters $a$, $b$, $m$ and $n$, we can obtain a finite element model by quadrilateral mesh shown in Figure 4(a). We apply the 5th percentile adult female among GEBOD Dummy Model shown as in the first right side of Figure 4(b) (LANCEMORE Corporation) developed in the airplane industry, because elder people and disabled people are assumed as an occupant. The model for tsunami pod made in the process mentioned as above is called the initial model without occupant restraint system in the following of this paper. The number of nodes and elements for each part of a pod, a rigid wall, and seawater (tsunami) are shown as in the Table 2. The
Figure 3. (a) Origami parts for Figure 2. (b) Side view of an origami ellipsoid of (a). This can be folded by pushing z-axis direction.

The total number for the dummy system is 36138 which are divided into 1636 for shell elements and 2648 for solid ones. The model which a laid occupant is imported into is shown as in the Figure 4(c).

Next, each part is divided into meshes for structure analysis. The way of mesh division is as follows; firstly each side composing a triangle is divided into 5, next those nodes are connected into square meshes regularly and triangle meshes in the case of including a key mode. And, the part of a rigid wall constructed of shell elements is divided into tetrahedron mesh, while the part of seawater constructed of solid elements in 400 times as large as a pod is divided into tetrahedron mesh shown as in Figure 5. The total process for simulation is from the moment when seawater (tsunami) shown as the right panel in Figure 5 starts flowing at an initial constant velocity to the moment when a pod rises onto a wall shown as in the left panel in Figure 5 after hitting on it.

2.4. Analysis theory: fluid-structure coupling analysis

In numerical simulation, we use the software “LS-DYNA” (LSTC Corp., 2014) which is a general-purpose finite element program capable of simulating complex real world problem. This numerical simulation is categorized as fluid structured coupling analysis where a fluid force transform a structure of pod and as well as a transformed structure influences a flow field. In this work, seawater and air are set
Figure 4. (a) A tsunami pod with quadrilateral mesh. (b) GEBOD Dummy models. (c) A model implemented with a dummy model.

Figure 5. Whole model for simulation including a pod, tsunami, and a rigid wall.

as fluid part, while tsunami pod is set as structured part. Those fluid parts are expressed in the Arbitrary Lagrangian Eulerian (ALE) method, and those structured parts are expressed in coordinate system. The ALE method is to solve simultaneously the conservation rules of mass, momentum, and energy. Each conservation rules are formally written as (9), (10) and (11).

\[
\frac{\partial \rho}{\partial t} + \rho \text{div}(\mathbf{v}) + (\mathbf{v} - \mathbf{w}) \cdot \text{grad}(\rho) = 0, \quad (9)
\]

\[
\rho \frac{\partial \mathbf{v}}{\partial t} + \rho (\mathbf{v} - \mathbf{w}) \cdot \text{grad}(\mathbf{v}) = \text{div}(\mathbf{\sigma}) + \mathbf{f}, \quad (10)
\]

\[
\rho \frac{\partial \mathbf{e}}{\partial t} + \rho (\mathbf{v} - \mathbf{w}) \cdot \text{grad}(\mathbf{e}) = \mathbf{\sigma} : \mathbf{D} + \mathbf{f} \cdot \mathbf{v}. \quad (11)
\]
<table>
<thead>
<tr>
<th>Part</th>
<th>Element type</th>
<th>The number of elements</th>
<th>The number of nodes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pod</td>
<td>Shell</td>
<td>4960</td>
<td>4957</td>
</tr>
<tr>
<td>Wall</td>
<td>Shell</td>
<td>3200</td>
<td>3281</td>
</tr>
<tr>
<td>Seawater (Tsunami)</td>
<td>Solid</td>
<td>18240</td>
<td>20454</td>
</tr>
<tr>
<td>Dummy model</td>
<td>Shell / Solid</td>
<td>1636 / 2648</td>
<td>7446</td>
</tr>
<tr>
<td><strong>Total number</strong></td>
<td></td>
<td><strong>26400</strong></td>
<td><strong>36138</strong></td>
</tr>
</tbody>
</table>

Table 2. Details for a whole model.

where $\rho$ is fluid density, $\mathbf{v}$ is velocity vector of fluid, $\mathbf{w}$ is velocity vector of mesh, $e$ is energy per unit mass, $\mathbf{\sigma}$ is Cauchy stress tensor, $\mathbf{D}$ is deformation velocity tensor, $\mathbf{f}$ is body force. In numerical simulation, seawater (fluid part) shown as in Figure 6(a) touches tsunami pod (structured part), and then the mesh of seawater and air are deformed internally (Figure 6(b)), and finally the mesh is restored to the initial position (Figure 6(c)). In this procession, the advection term including $\mathbf{v} - \mathbf{w}$ in (11) is calculated [1].

![Figure 6. The Arbitrary Lagrangian and Eulerian (ALE) method. (a) Initial mesh. (b) Deformed internal mesh. (c) Mesh restored to the initial position.](image)

The applied penalty method generally used in the contact calculation for structure analysis is applied into coupling calculation method with structured part (pod) and fluid (tsunami). As shown in Figure 7, those contact points are placed on the contact positions with structure part and fluid one. The contact points are shifted following to the fluid transfer, however the virtual spring between the transferred point and the former one is generated. The force exerted by the extended spring expresses the pressure which a structure is exerted by fluid. The spring stiffness $k$ for the virtual string is given as (12).

$$
 k = p_f \frac{K A^2}{V}, 
$$

where $K$ is a volume modulus for a fluid element, $A$ is average area for a structure element, $V$ is a volume for a fluid element, $p_f$ is scale factor [1]. On this condition,
the force $P$ which the structure body receives is expressed as (13).

$$P = kd,$$

(13)

where $d$ is intrusion of fluid into structure body. This coupling method has short time steps for an explicit method, therefore intrusion into fluid for 1 step is a little.

![Fluid structure coupling procedure using penalty based method. (a) Initial configuration. (b) Generation of artificial spring at the next step.](image)

**2.5. Conditions for simulation and video image shots**

CFRP applied into a pod is compound material which has a layered structure. As a product specification for a commercial CFRP, a tensile strength and a bending stress (nominal stress) are mentioned. Therefore, in this paper, a pod is dealt as a simple elastic body, and the load provided on its structure is estimated by von Mises stress to compare with the strength of an actual product. The thickness of a tsunami pod in simulation is set as 4.0 mm which is the minimum among the products. The properties for CFRP as material are as follows; Young’s modulus is 294 GPa, Poisson’s ratio is 0.12, and density is $1.8 \times 10^{-9}$ ton/mm$^3$. A tsunami velocity is calculated as follows. A tsunami velocity in transmitting on the sea is in proportion to the root of sea depth, however a tsunami velocity we apply into simulation is the one after flowing into the land which is slower than to the one on the sea, because there are factors of reducing velocity that depth is lower and there are obstacles. The velocity for simulation is calculated as 8 km/sec by the distance which certain object move for a given period in the video taken in the Tohoku earthquake. Figure 8 shows video image shots in numerical simulation from the moment tsunami touches a pod to the moment the tsunami pod rises onto the wall. In the latter process of this numerical simulation, an occupant hits on the internal wall of a tsunami pod to be injured possibly. For preventing that, we try to reinforce an occupant with an occupant reinforcement system similar to a safety bar applied into roller coaster. In the next section, the strength of a tsunami pod
and the injury degree of an occupant are compared between the model without a reinforcement and the one with it.

![Video image shots in numerical simulation.](image)

3. Analysis result of numerical simulation

3.1. Strength confirmation of the initial model without an occupant restraint system

Firstly, strength is verified in case of the initial model without an occupant reinforcement system. It would be evaluated as not satisfying safety, if any elements of a pod exceeded von Mises equivalent stress for CFRP (500 – 1500 MPa). As a result of the numerical simulation, the maximum of von Mises equivalent stress value among each element (written shortly as Maximum Mises stress in the following sentences) is 1272 as illustrated in Figure 9, it is judged that a pod may be deformed or broken.

![Color contour for the maximum of von Mises equivalent stress at the moment that a resultant force become maximum in case of the initial model without an occupant restraint system.](image)

3.2. Confirmation of Head Injury Criterion for the initial model

Head Injury Criterion (HIC) is a measure of the likelihood of head injury arising from an impact. HIC is calculated by (14) using head acceleration and its duration. A big acceleration value for a short duration period is allowed, because the calculated HIC value is an average of every 0.036 seconds. The value below
1000 is judged as safe where 1 of 6 people suffer a critical injury in the brain. The time transformation of HIC in case of the initial model is shown as in Figure 10. It is proved that the value calculated by (14) is 8142 which is around 8 times as the safety level 1000, therefore it is likely that an occupant would be seriously damaged.

\[
HIC = \left\{ \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} a(t) \, dt \right\}^{2.5} (t_2 - t_1),
\]

Figure 10. Resultant acceleration for the head of an occupant in case of the initial model without an occupant restraint system.

where \( t_1 \) and \( t_2 \) indicate the initial and final times (seconds) and \( a \) indicates acceleration. Here we suppose that \( |t_2 - t_1| \leq 0.036s \).

3.3. Strength confirmation of the modified model with an occupant reinforcement system and its HIC value

It is confirmed from the video image shots as above that an occupant is rotating violently and then hits on the internal side of a pod, finally is injured in the head severely. For an automobile, 4-point or 5-point seat belts are adopted to respond to front collision, the reinforcement to fix an upper body and a lower body separately which imitates the safety bars of a roller coaster is designed as Figure 11. Then, is implemented with the designed bars, the seat, and a dummy model whose positions are arranged whose position is arranged not to contact with other parts as Figure 11(b). This model is called model with an occupant restraint system in the following. The parts of a seat and a reinforcement are integrated with a pod to be defined as a rigid body consisting of 5914 nodes and 5858 shell elements totally. The thickness of an outer wall is increased from 4.0 mm in case of the initial model.
to 6.0 mm for strengthening its structure. As a result of the numerical simulation in case of the modified model with an occupant reinforcement system, maximum Mises stress in some elements of a pod in collision is 789 MPa which is less than the strength of CFRP (500 – 1500 MPa) and the condition for damage failed to be improved shown as in Figure 12. On the other hand, HIC value is decreased by more than 90% to become 76 in value shown as in Figure 13 which is well below the safety level 1000.

4. Optimization of structure of tsunami pod

4.1. Conditions for optimization

By applying the modified model with a reinforcement, the following optimization is attempted. Those ranges for each design variable are as follows:
Figure 13. Resultant acceleration for the head of an occupant in case of the model with an occupant restraint system.

900 < major axis($a$) < 1320, 600 < minor axis($b$) < 880, 2.0 < thickness($t$) < 6.0, 345 < Young’s modulus($E$) < 450. Here, HIC is excluded from consideration, because it has already satisfied safety in the result of the simulation applying the modified model with a reinforcement. Under the above conditions, each design variables are calculated to minimize the mass of a pod. The velocity is set as constant 4 m/sec in the $x$ axis. For shortening a calculation time, a dummy model is set as rigid body, and apply decoupling analysis in optimization, and later the validity of the obtained optimal model is verified by coupling analysis. Incidentally, the calculation time for 1 step in decoupling analysis is one half as short as the one in coupling analysis. We use a software named LS-OPT [1] applying Radial Basic Function (RBF) as response surface function. Initially, 5 sample points are input and an optimum solution is searched while generating response surfaces of a objective function and a constraint function by RBF network. The condition of finishing a calculation is the case where the relationship of optimal value obtained this step and the one obtained in the last step. If (15) is not satisfied, 5 sample points are added.

\[
\frac{\text{optimal value obtained in this step} - \text{optimal value obtained in the last step}}{\text{optimal value obtained in this step}} \leq 0.01.
\] (15)

4.2. Result of optimization

Firstly, we check the correlation between each design value, objective values and the value of constraint condition as in the Table 4. It is found that mass of pod is
Design value

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Major axis (mm)</td>
<td>$900 &lt; a &lt; 1320$</td>
</tr>
<tr>
<td>Minor axis (mm)</td>
<td>$600 &lt; b &lt; 880$</td>
</tr>
<tr>
<td>Thickness (mm)</td>
<td>$2.0 &lt; t &lt; 6.0$</td>
</tr>
<tr>
<td>Young’s modulus (GPa)</td>
<td>$345 &lt; E &lt; 450$</td>
</tr>
</tbody>
</table>

Objective function

Minimizing the mass of pod (kg)

Constraint condition

Maximizing von Mises equivalent stress (MPa) for each element is less than the strength for CFRP

Table 3. The range of design value, objective function and constraint condition. In this table, the mark * implies that HIC is excluded.

Higher positive correlated with a major axis and a minor axis than with thickness and Young’s modulus, so it is necessary to reduce a size for making the model of a pod light-weighted. And, also it is found that thickness needs to be smaller and Young’s modules needs to be lower. While the maximum Mises stress as constraint condition has a higher negative correlation with major axis and minor axis than Young’s modules, and little correlated to thickness. It can be said for minimizing the maximum Mises stress it is necessary to increase a size and Young’s modules. Therefore, as a result of the above consideration, optimization is predicted to be under unstable process in changing the size while arranging thickness and Young’s modules.

The relationships are illustrated as in Figure 14 (a), (b) and (c) drawn by a software for optimization “LS-OPT” with an interface to LS-DYNA. These make it easier to understand the relationship as mentioned above visually.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Major axis (mm)</th>
<th>Minor axis (mm)</th>
<th>Thickness (mm)</th>
<th>Young’s modulus (GPa)</th>
<th>Mass (kg)</th>
<th>Maximum Mises (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variables</td>
<td>-</td>
<td>0.97</td>
<td>-0.17</td>
<td>0.29</td>
<td>0.83</td>
<td>-0.90</td>
</tr>
<tr>
<td>Major axis (mm)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.29</td>
<td>0.88</td>
<td>-0.90</td>
</tr>
<tr>
<td>Minor axis (mm)</td>
<td>-</td>
<td>-</td>
<td>-0.16</td>
<td>0.29</td>
<td>0.88</td>
<td>-0.90</td>
</tr>
<tr>
<td>Thickness (mm)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-0.07</td>
<td>0.16</td>
<td>0.05</td>
</tr>
<tr>
<td>Young’s modulus (GPa)</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.23</td>
<td>-0.17</td>
</tr>
</tbody>
</table>

Table 4. The values of correlation coefficient between variables.
The result of optimization process is shown as in Table 5 which illustrates those pod images during optimization process from time step 1 to 6. The calculation takes 5 days to run from step 1 to step 6. And, in checking the result, the optimization of size is converged in the 4th step, and after that thickness and Young’s modules are made to be converged until 6th step. The comparison between the optimal model from 6th step and the initial model is shown as in Table 6.

<table>
<thead>
<tr>
<th>times of run</th>
<th>a (scale)</th>
<th>b (actual)</th>
<th>b (scale)</th>
<th>h (mm)</th>
<th>Young’s modules (GPa)</th>
<th>mass (kg)</th>
<th>constraint condition (max von mises(MPa))</th>
<th>image of model</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.00</td>
<td>1205</td>
<td>0.75</td>
<td>600</td>
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<td>240</td>
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Table 5. Optimization process from time step 1 to 6.

Acknowledgments. The authors would like to thank referees for helpful comments and suggestions. This work was partially supported by JSPS KAKENHI Grant Number 15H02228.
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<td>Minor axis (mm)</td>
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<td>Thickness (mm)</td>
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<td>Young’s modulus (GPa)</td>
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Table 6. Comparison between the initial values before optimization and the optimal values after optimization.

References

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