A Study on the Optimum Bucket Size for Master Scheduling:
For the Case of Hierarchically Structured Products

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Abstract

The function of master scheduling is to plan the flow of order from its arrival to its completion. In this study, the problem of bucket size for master scheduling is taken up. The bucket size for master scheduling has much influence on the lead time of the order. However, to date there is no clear method for how to set the optimum bucket size. The purpose of this study is to propose a method to set the optimum bucket size. In this paper, an equation to estimate the optimum bucket size is proposed for the case where a production system and products have hierarchical structure.

Key Words: bucket size, master schedule, periodic scheduling method, lead time, loading, hierarchically structured production system, products with tree structure

1. Introduction

Today, all manufactures have to shorten the manufacturing lead time. And manufacturers which adopt a make-to-order production system have to make many kind of products, while the specification of product differs from order to order. This situation requires an efficient master scheduling system which takes the contents of the order and the capacity of production system into consideration. The function of master scheduling is to plan the flow of order from its arrival to its completion.
This study focuses on the bucket size for a master scheduling system. The bucket size influences the manufacturing lead time. However, how the optimum bucket size can be set has not been made clear up to now. Fig. 1 shows the relationship between manufacturing lead time and bucket size\(^1\).

Fig. 2 shows the influence of bucket size on lead time in the case where all products flow from planning shop No. 1 to planning shop No. 2. When the bucket size is too small, the lead time is very long, because all the jobs which arrive at the planning shop can not be loaded in one bucket, and some of them overflow to the next bucket (see Fig. 2–b). However, when the bucket size is too large, the lead time is also long (see Fig. 2–c). Accordingly, there is a bucket size which makes the lead time of the order shortest (see Fig. 2–a)\(^2\).

The purpose of this paper is to propose a method to set the optimum bucket size for the case where the production system and the products are hierarchically structured. First, the production system and work are defined, and the simulation model is constructed. Secondly, the experiments are conducted and the behavior of the optimum bucket size is made clear. Lastly, an equation to estimate the optimum bucket size is proposed.

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\(^1\) A Study on the Optimum Bucket Size for Master Scheduling

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\(^2\) Figure 2 Influence of bucket size on lead time
2. Definition

In the real world, production system and work have a hierarchical structure such as shown in Fig. 3. The number of hierarchies differs company by company according to the structure of the production system, the complexity of the products, and the complexity of the routing to make the products.

In this study, a production system and work are defined as follows.

(1) **Work Station and Operation**

The work station (W/S) is the basic unit of the production system. An operation corresponds to a work station, that is, one operation is executed at one work station without interruption.

(2) **Work Center**

A work center (W/C) is a set of work stations.

(3) **Shop**

A shop is a set of work centers.

(4) **Planning Shop and Job**

A planning shop (P/S) is composed of one or several shops. A job, which is a set of operations, corresponds to a planning shop. The set of operations performed at the work station within the same planning shop successively is a job. A master schedule is made by loading jobs to the planning shop (see Fig. 4).

(5) **Factory and Order**

A factory is the whole of the production system and the set of planning shops. An order, which is a set of jobs, corresponds to a factory. The set of jobs to be done in the factory is an order.

![Figure 3 Production system and work](image-url)
A Study on the Optimum Bucket Size for Master Scheduling

3. Model of the research

In this paper, the model as shown in Fig. 5 was constructed. A factory is composed of several planning shops. All planning shops are composed of only one shop. A shop is composed of only one work center. A work center is composed of several work stations. The routing of a job in a planning shop is the flow type. The composition of all planning shops is the same. The number of planning shops in level 2 is from one to eight. The number of planning shops in level 1 is one. In this paper, the factory where the number of planning shops in level 2 is one is called model type S (a single planning shop). The factory in which the number of planning shops in level 2 is from two to eight is called model type M (multiple planning shops).
In this model, the flow of work within the production system is as follows. In the case of model type S, the order arrives at the factory randomly. Its arrival time at the planning shop in level 2 equals the order’s arrival time at the factory. The arrival time at the planning shop in level 1 is equal to the time when all the jobs charged to the planning shop in level 2 are completed. A master schedule is made using a loading method. This schedule is sent to the production system. In the production system, dispatching is done based on this schedule, and operations are executed. In the case of model type M, the order also arrives at the factory randomly. The arrival time at all planning shops in level 2 equals the order’s arrival time. The arrival time at the planning shop in level 1 is equal to the time when all jobs charged to the planning shops in level 2 are completed. The details of the models are as follows.

3.1 Master scheduling

A master schedule is made by loading jobs in the bucket. Now suppose that the loading of the \( (N+2) \)th bucket is to be done at time point \( t \) (see Fig. 6-a). Jobs waiting to be loaded at a certain planning shop can be classified into three groups. The first group (three jobs), marked by white, have already been started but not completed. The second group (one job), marked by gray, is already loaded but not started. The third group (the other jobs), marked by black, have not been loaded. Some or all of these jobs are loaded to the \( (N+2) \)th bucket.
according to priority. In this paper, the FCFS rule is adopted to determine the priority. As a loading method, finite loading is adopted. In Fig. 6-a, job 1 to job 7 can be loaded to the \((N+2)\)th bucket, but job 8 cannot be loaded. The priority rule of dispatching is the order in which the jobs are loaded.

It is assumed that a breakdown of the production system never happens, and the estimated processing time has no error.

![Diagram](image)

Figure 6  Flow of work from master scheduling to execution of operation

3.2 Factors taken up in the model

In this paper, the factors shown in Table 1 are taken up. The capacity of a planning shop is given by equation (1).

\[ C = b \times m \]  

Where

- \(C\) : capacity of a planning shop
- \(b\) : bucket size
- \(m\) : number of work stations

The ratio of the total work to arrive at the planning shop to the capacity of the planning shop is called arrival load ratio in this paper. Arrival load ratio is decided by relationship between the factors concerned with production system and the factors concerned with work.
Almost all manufacturers want to make the manufacturing lead time as short as possible. Therefore, as a criterion of the optimum bucket size, mean lead time is adopted. Lead time of the order is the time period between its arrival time at the factory and its completion time. So mean lead time is expressed as equation (2).

\[ L = \frac{\sum l_i}{n} \]  

(2)

Where

- \( L \) : mean lead time
- \( l_i \) : lead time of order \( i \)
- \( n \) : number of sample

3.4 Approach

As this paper takes up a dynamic situation, it seems that an analytical approach cannot be taken. Therefore, a simulative approach is adopted.

4. Experimental results

An example of the relationship between bucket size and mean lead time in the case of model type M is shown in Fig. 7. As shown in Fig. 7, when the bucket size is too small, the mean lead time is infinite, because all jobs which arrive at planning shop can not be finished, therefore the mean lead time increases as time goes on. As the bucket size becomes larger, all the jobs which arrive at the planning shop can be done, the mean lead time becomes shortest when the bucket size is a certain length. From that point, the mean lead time increases gradually. It has been found that a bucket size which makes mean lead time shortest exists. The result of the same investigation of model type S and model type M found that this is true regardless of the number of planning shops in level 2, the number of work stations and

<table>
<thead>
<tr>
<th>Table 1 Factors taken up in the model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factor</td>
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<tr>
<td>Production system</td>
</tr>
<tr>
<td>number of work stations ((m))</td>
</tr>
<tr>
<td>number of planning shops in level 1 ((p1))</td>
</tr>
<tr>
<td>number of planning shops in level 2 ((p2))</td>
</tr>
<tr>
<td>number of levels ((pl))</td>
</tr>
<tr>
<td>Work</td>
</tr>
<tr>
<td>number of operations ((o))</td>
</tr>
<tr>
<td>number of jobs ((j))</td>
</tr>
<tr>
<td>processing time ((e))</td>
</tr>
<tr>
<td>routing of job ((r))</td>
</tr>
<tr>
<td>order arrival pattern to the factory ((v))</td>
</tr>
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<td>Arrival load ratio ((\rho))</td>
</tr>
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</table>
the arrival load ratio. In this paper, the optimum bucket size is defined as the bucket size where the mean lead time is shortest.

5. **Equation to estimate the optimum bucket size**

An equation to estimate the optimum bucket size is made as follows.

5.1 **Equation to estimate the optimum bucket size in the case of model type S**

Fig. 8 shows the relationship between the optimum bucket size and the number of work stations in the case of model type S.
As shown in Fig. 8, the optimum bucket size increases linearly according to the number of work stations. Therefore, the optimum bucket size in case of model type S can be written as equation (3),

\[ B_S = am + b \]  

(3)

Where
- \( B_S \): optimum bucket size in case of model type S
- \( a \): gradient
- \( m \): number of work stations
- \( b \): constant

From the result of the experiment, the gradient (a) of equation (3) and the constant (b) of equation (3) are found by the least square method. Fig. 9 shows the relationship between gradient (a) of equation (3) and arrival load ratio.

As the arrival load ratio approaches 1.0, the gradient increases exponentially. If the arrival load ratio equals one, the system never reaches a steady state, therefore, the gradient is infinite. Then, the gradient can be expressed as equation (4).

\[ a = f \times \frac{1}{(1 - \rho)} \]  

(4)

Where
- \( \rho \): arrival load ratio
- \( f \): coefficient

The coefficient (f) of equation (4) is found by curve fitting. Fig. 10 shows the relationship between arrival load ratio and coefficient (f) of equation (4).
As was shown in Fig. 10, the relationship between coefficient $(f)$ and arrival load ratio is a straight line. Therefore, equation (5) is obtained.

$$f = 3.2156\rho - 1.8859$$  \hspace{1cm} (5)

Then,

$$a = \frac{3.2156\rho - 1.8859}{(1-\rho)}$$  \hspace{1cm} (6)

Applied same method, constant $(b)$ of equation (3) can be written as equation (7).

$$b = \frac{-3.4256\rho + 3.4504}{(1-\rho)}$$  \hspace{1cm} (7)

Due to the results mentioned above, the equation to estimate the optimum bucket size in the case of model type S is expressed as follows.

$$B_s = \frac{3.2156\rho - 1.8859}{(1-\rho)}m + \frac{-3.4256\rho + 3.4504}{(1-\rho)}$$  \hspace{1cm} (8)

5.2 **Equation to estimate the optimum bucket size in the case of model type M**

In the case where the number of work stations in planning shops is three, the relationship between the optimum bucket size and the number of planning shops in level 2 is shown in Fig. 11.

As was shown in Fig. 11, the optimum bucket size is constant regardless of the number of planning shops in level 2. Fig. 11 shows the result in the case where the number of work stations is three, however when investigating of other conditions, it is found that this is true regardless of the number of work stations. Therefore, the equation to estimate the optimum bucket size in the case of model type S can be applied to model type M.
5.3 Precision of equation

Table 2 shows the difference between the optimum bucket size estimated by equation (8) and that of the experiment in the case where the number of planning shops in level 2 is one. As was shown in Table 2, the ratio of error is very low. As a result of the investigation of other conditions, this is true regardless of the number of planning shops in level 2. Due to the result mentioned above, equation (8) gives a fairly correct estimation.

<table>
<thead>
<tr>
<th>arrival load ratio</th>
<th>number of work stations</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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<tbody>
<tr>
<td>0.75</td>
<td>5%</td>
<td>1%</td>
<td>4%</td>
<td>3%</td>
<td></td>
</tr>
<tr>
<td>0.80</td>
<td>-7%</td>
<td>-3%</td>
<td>2%</td>
<td>-5%</td>
<td></td>
</tr>
<tr>
<td>0.85</td>
<td>-6%</td>
<td>-2%</td>
<td>2%</td>
<td>-6%</td>
<td></td>
</tr>
<tr>
<td>0.90</td>
<td>1%</td>
<td>1%</td>
<td>6%</td>
<td>0%</td>
<td></td>
</tr>
</tbody>
</table>

6. Conclusion

The purpose of this paper is to propose a method to set the optimum bucket size in the case where the production system and products are hierarchically structured. 

Firstly, a simulation model for a hierarchically structured production system and products with a tree structure was built. Secondly, using this model, experiments were conducted. Due to the behavior of lead time against the bucket size, it was found that a bucket size exists which makes the lead time shortest. This bucket size was defined as the optimum bucket size. Thirdly, an equation to estimate the optimum bucket size in the case where the production
system and products have a tree structure was deducted. And then the relationship between
the optimum bucket size and the number of planning shops in level 2 was made clear.

The future direction of the study will need to take up the case where the number of
hierarchies is more than two. And where the factors, such as a variance of processing time,
the number of product structure, the set-up time of jobs and so forth, should be taken up.

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