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Applicability of Actuarial Method to Option Pricing

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Abstract

We studied the mathematical structure of option pricing both in the Black-Scholes model and the Actuarial model. In the two models, the option prices for the Construction Bank of China, for example, are calculated. Applicability of the two models to the market option prices is discussed. Comparing the numerical results of the option prices calculated in the two models, we discuss how the actuarial model predicts option prices more precisely than the Black-Scholes model.

Key Words: Option Pricing, Black-Scholes Formula, Actuarial Method

1. Introduction

Buying an option is one of the most successful trading methods introduced to hedge risk in investment and other risky trades. When one buys an option, he has to pay the option price. This cost gives rise to a secondary risk, even though the option is traded to hedge risk for investment. Option pricing is a significant task to make the risk hedge effective.

Black and Scholes[1] invented a formula for option pricing, which is now widely used in finance, investment and other fields in the market. In deriving their formula for the value of an option in terms of the price of the stock, they assume ideal conditions in the market for the stock and for the option.

The European call option price is determined to be equal to a mathematical expectation of the difference of the stock price from the option exercise price at the expenditure date[2]. The Black-Scholes option price derived from a risk-neutral assumption is free from the expected return rate of the stock. However, the buyers of the option may expect a rising variation of the stock price through time, when they buy a call option. The expectation for a rising stock price brings about a higher option pricing in the market than the Black-Scholes value of the option price.

Merton[3] introduced the Black-Scholes pricing model to the field of insurance. Bladt and Rydberg firstly turned the pricing problem into an equivalent insurance or fair premium determination in 1998. Their actuarial method has been used to solve the option pricing problem.

Actuary[4] is one field of applied mathematics that uses mathematical models to estimate and analyze the influences brought by the risk of uncertain events in the future. Financial problems related to uncertainty can be analyzed and solved by using actuarial ideas. This fact provides a theoretical basis to the actuarial methods which are used to solve the problem of option pricing.

Option trading is a deal that is related to insurance. The option buyer is equivalent to the insured who pays for a certain premium in order to gain the potential benefits, and the option seller is equivalent to the insurer who gets premium, bearing the risk. An option contract is essentially an insurance contract between the insured and the insurer. The insurance premium is a fair premium. The actuarial fair price principle to determine the premiums charged by insurers may be used to calculate the fair price for the premium of the call option.

Insurance is characterized by determining the price before any accidental losses happen. In an actuarial model, the insurers usually use an actuarial equivalence principle, when they determine the insurance premium. The principle means that the revenue and the future expenditure should be balanced in the actuarial present value, that is, the insurance premium is determined to be the present value of compensation for future loss. Option pricing is performed with the same idea, that is, an option premium is the present value of the buyer's future earnings from the option. When an option is purchased, the option premium is determined to be the value of the difference of the maturity price from the strike price of the stock. The option price is calculated by taking into account the probability distribution of stock prices.

The Actuarial method is different from the Black-Scholes Martingale pricing method for options in the probability measurement used to calculate the mathematical expectation of the exercised option price and the possible loss. In the Martingale approach, the European call option price is equal to a mathematical expectation of the difference of the expenditure stock price from the option exercise price in the equivalent Martingale measure. The equivalent Martingale probability measure is the process of discounting the stock price in a Martingale probability measurement, which usually is not the actual probability measure of the stock price. When the financial market is arbitrage, non-equilibrium (no equivalent Martingale measures exist) or incomplete (an equivalent Martingale measure exists, but is not unique), the Martingale method cannot be applied[6]. The actuarial method converts option pricing problem in determining an equivalent fair premium price. Because no economical assumptions are taken into account, the method is effective not only in a non-arbitrage, equilibrium and complete market, but also in an arbitrage, non-equilibrium and incomplete market. The actuarial approach uses a contingent claim structure of an option. Therefore, the approach cannot be carried over immediately to general derivative security pricing. Moreover, no economical considerations are involved other than discounting factors.

The Black-Scholes[7] pricing model uses the portfolio replication method, which assumes that investors can continuously and finely adjust the hold position of the stock and the option in order to get a risk-free portfolio. But, in the real market, adjusting the portfolio infinitely in transient time cannot be realized. This paper, relaxing the risk-free assumption, studies the actuarial option pricing model, from the viewpoint of the evaluation of the seller's potential losses which resulted from the implementation of options and the corresponding probability distribution of the stock prices. When the stock price is rising in the market over time, the actuarial model correspondingly sets a higher option price. The actuarial model often gives a better prediction for an option price than the Black-Scholes formula. In this paper, we will make clear the difference of the option prices between the Black-Scholes model and the Actuarial model. The applicability of these models to the option prices in market is discussed.

2. Option Pricing Models

1) *Black-Scholes formula*

In terms of the expected return rate μ and volatility σ, the return rate of the stock is expressed as

$$\frac{dS(t)}{S(t)} = \mu dt + \sigma dz(t) \tag{1}$$

and the option price $C(t)$ is assumed to satisfy the same equation,

$$\frac{dC(t)}{C(t)} = \mu_C(S(t), t)\, dt + \sigma_C(S(t), t)\, dz(t) \tag{2}$$

The probability fluctuations $dz(t)$ take place in infinitesimal time interval dt as

$$\{dz(t)\}^2 = dt. \tag{3}$$

Expressing the option price as $C(t) = f(S(t), t)$, we obtain the variation of the option price $C(t)$ in time interval dt,

$$dC(t) = f_S(S(t), t)\, dS(t) + \frac{f_{SS}(S(t), t)}{2}\{dS(t)\}^2 + f_t(S(t), t)\, dt. \tag{4}$$

Owing to Eqs. (1) and (3), in the above equation in first order in dt, we must adopt the term in order of

$$\{dS(t)\}^2 = \{\sigma S(t)\}^2 dt. \tag{5}$$

Substituting eq. (5) into eq. (4), we obtain the equation for the variation of the option price $C(t)$,

$$\begin{aligned} dC(t) &= f_S(S(t), t)\, S(t)(\mu dt + \sigma dz(t)) + \frac{f_{SS}(S(t), t)}{2}\{\sigma S(t)\}^2 dt \\ &\quad + f_t(S(t), t)\, dt \\ &= \left[\mu f_S(S(t), t)\, S(t) + \frac{f_{SS}(S(t), t)}{2}\{\sigma S(t)\}^2 + f_t(S(t), t)\right] dt \\ &\quad + \sigma f_S(S(t), t)\, S(t)\, dz(t). \end{aligned} \tag{6}$$

Thus, the return rates $\mu_C(S(t), t)$ and $\sigma_C(S(t), t)$ for the option are expressed as

$$\begin{aligned} \mu_C(S(t), t) &= \frac{1}{f(S(t), t)}\Big[f_t(S(t), t) + f_S(S(t), t)\mu S(t) \\ &\quad + \frac{f_{SS}(S(t), t)}{2}\sigma^2 S^2(t)\Big], \\ \sigma_C(S(t), t) &= \frac{f_S(S(t), t)\sigma S(t)}{f(S(t), t)} \end{aligned} \tag{7}$$

with the partial derivative f_t.

The option pricing of Black and Scholes is specialized by risk-neutralization: they assume that the buyer of the option takes a stance for risk-neutralization to make a portfolio of buying the option and selling the stock. Therefore they have the risk price of the option equal to that of the stock[8],

$$\frac{\mu - r}{\sigma} = \frac{\mu_C(S(t), t) - r}{\sigma_C(S(t), t)} \tag{8}$$

with the risk-free interest rate r. Substituting eq. (7) into eq. (8), we obtain the Black-Scholes equation for the option price,

$$rSf_S(S, t) + f_t(S, t) + \frac{\sigma^2 S^2}{2} f_{SS}(S, t) = rf(S, t). \tag{9}$$

Analytically the Black-Scholes equation is solved in a same way as a thermal diffusion equation. With the boundary condition

$$C(T) = \mathrm{Max}(S_T - K, 0), \tag{10}$$

the Black-Scholes equation is solved. The solution is as follows,

$$C(t) = f(S, t) = S\Phi(d) - K_e^{-r(T-t)}\Phi(d - \sigma\sqrt{T-t}) \tag{11}$$

with

$$d = \frac{\log(S/K) + r(T-t)}{\sigma\sqrt{T-t}} + \frac{\sigma\sqrt{T-t}}{2},\ \Phi(d) = \int_{-\infty}^{d} \frac{1}{\sqrt{2\pi}} e^{\frac{-\mu^2}{2}} d\mu.$$

2) *Actuarial model*

We discuss simply the European call option. The underlying asset is assumed to be a common stock. The following notations are used: $C(S(t), t)$ is the European call option price at moment t, K is the option strike price, the expiration date is T, the underlying stock price at moment t is S_t, $t \in [0, T]$, and S_T is the stock price at maturity.

Insurance is characterized by the feature that pricing is taking place before any loss happens, so there is a time difference between the insurer charging premiums and paying damages. In order to calculate potential future losses, it is necessary to discount the maturity stock price and the strike price to the initial moment values[9]. According to call option trading, only if the stock price is greater than the strike price at maturity, will the option be implemented[10]. The corresponding present value of the insurer's loss is $e^{-\mu(T-t)} S_T - e^{-r(T-t)}K$. The maturity price S_T of the stock is discounted by the expected return rate μ, and the strike price K is discounted by the risk-free interest rate r.

According to the pricing models, the option price is determined by the probability that the final stock price S_T rises above the implementation price K in circumstances where the current stock price is given[11]. The actuarial model supposes that the stock price $S(t)$ follows the Geometric Brownian motion whose drift rate (expected return rate) is a constant μ and volatility rate is a constant σ. Then, the distribution of stock price $S(t)$ is expected to follow the lognormal distribution. That is,

$$\log S(t) \sim N\left(\left(\mu - \frac{\sigma^2}{2}\right)t, \sigma^2 t\right).$$

Under the supposition of Brownian motions for the stock price, the actuarial option pricing model derives the probability density function of the stock price $S(T)$ at maturity. The securities return rate is expressed as

$$\frac{dS(t)}{S(t)} = \frac{S(t+dt)-S(t)}{S(t)} = \mu dt + \sigma dz(t). \tag{12}$$

In infinitesimally short time period dt, the expected value and variance of the securities prices are

$$E\left[\frac{S(t+dt)-S(t)}{S(t)}\right] = \mu dt$$

$$V\left[\frac{S(t+dt)-S(t)}{S(t)}\right] = E\left[\left(\frac{S(t+dt)-S(t)}{S(t)} - \mu dt\right)^2\right] = \sigma^2 dt. \tag{13}$$

Since

$$\log S(t+dt) - \log S(t) = \log\frac{S(t+dt)}{S(t)} = \log\left\{1+\frac{S(t+dt)-S(t)}{S(t)}\right\}$$
$$= \frac{S(t+dt)-S(t)}{S(t)} - \frac{1}{2}\left\{\frac{S(t+dt)-S(t)}{S(t)}\right\}^2 + \ldots, \tag{14}$$

the expected value and variance of the logarithmic return rate in dt imply that

$$E[\log S(t+dt) - \log S(t)] = \mu dt - \frac{1}{2}\sigma^2 dt,$$
$$V[\log S(t+dt) - \log S(t)] = \sigma^2 dt. \tag{15}$$

Therefore, in order to take the stochastic fluctuations of securities prices $S(t)$ into account, the actuarial pricing model assumes that $\log S(t)$ follows the normal distribution $N\left(\log S(0) + \left(\mu - \frac{1}{2}\sigma^2\right)t, \sigma^2 t\right)$. Then, the probability density function of $\log S(t)$ implies that

$$f(\log S(t)) = \frac{1}{\sqrt{2\pi\sigma^2 t}} e^{-\frac{\left[\log S(t) - \log S(0) - \left(\mu - \frac{\sigma^2}{2}\right)t\right]^2}{2\sigma^2 t}}. \tag{16}$$

The above density function satisfies eq. (15) for finite time interval t:

$$E[\log S(t) - \log S(0)] = \int_{-\infty}^{\infty} \frac{\{\log S(t) - \log S(0)\}}{\sqrt{2\pi\sigma^2 t}} e^{-\frac{\left[\log S(t) - \log S(0) - \left(\mu - \frac{\sigma^2}{2}\right)t\right]^2}{2\sigma^2 t}} d\log S(t) = \left(\mu - \frac{\sigma^2}{2}\right)t,$$

$$V[\log S(t) - \log S(0)] = E\left[\left\{\log S(t) - \log S(0) - \left(\mu - \frac{\sigma^2}{2}\right)t\right\}^2\right]$$
$$= \int_{-\infty}^{\infty} \frac{\left\{\log S(t) - \log(0) - \left(\mu - \frac{\sigma^2}{2}\right)t\right\}^2}{\sqrt{2\pi\sigma^2 t}} e^{-\frac{\left[\log S(t) - \log S(0) - \left(\mu - \frac{\sigma^2}{2}\right)t\right]^2}{2\sigma^2 t}} d\log S(t) = \sigma^2 t. \tag{17}$$

In order to verify the stochastic fluctuations of securities prices in a short time period dt, we use the expansion,

$$\frac{S(t+dt)-S(t)}{S(t)} = e^{\log\frac{S(t+dt)}{S(t)}}-1$$
$$= \log S(t-dt)-\log S(t)-\left(\mu-\frac{1}{2}\sigma^2\right)dt\Big\}^2+\ldots$$
$$= \log S(t+dt)-\log S(t)\frac{1}{2}\Big\{\log S(t+dt)-\log S(t)$$
$$-\left(\mu-\frac{1}{2}\sigma^2\right)dt\Big\}^2+\{\log S(t+dt)-\log S(t)\}\left(\mu-\frac{\sigma^2}{2}\right)dt$$
$$-\frac{1}{2}\left(\mu-\frac{1}{2}\sigma^2\right)(dt)^2+\ldots$$

The expected value and variance of the securities return rate reproduce eq. (13):

$$E\left[\frac{S(t+dt)-S(t)}{S(t)}\right] = \left(\mu-\frac{1}{2}\right)dt+\frac{\sigma^2}{2}dt = \mu dt,$$
$$V\left[\frac{S(t+dt)-S(t)}{S(t)}\right] = \sigma^2 dt. \tag{18}$$

The option price should be equal to the mathematical expectation of the potential losses caused by striking the option. The formula for the option price implies that

$$C(S(t),t) = E\left(e^{-\mu(T-t)}S_T-e^{-r(T-t)}K\right)$$
$$= \int_{-\infty}^{+\infty}\left(e^{-\mu(T-t)}S_T-e^{-r(T-t)}K\right)f(\log S_T)d(\log S_T). \tag{19}$$

Only when the expiration stock price $S(t)$ is greater than the strike price K, the call option will be implemented. Eq. (19) is modified as

$$C(S(t),t) = \int_{\log K}^{+\infty} e^{-\mu(T-t)}S_T f(\log S_T)d(\log S_T)$$
$$-\int_{\log K}^{+\infty} e^{-r(T-t)}Kf(\log S_T)d(\log S_T). \tag{20}$$

Substituting eq. (16) into eq. (20), we obtain the option price

$$C(S(t),t) = S_t N(d_1)-Ke^{-r(T-t)}N(d_2), \tag{21}$$

where $N(.)$ is the standard normal cumulative distribution function with

$$d_1 = \frac{\log(S_t/K)+\left(\mu+\frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}},$$
$$d_2 = \frac{\log(S_t/K)+\left(\mu+\frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}} = d_1-\sigma\sqrt{T-t}. \tag{22}$$

If, according to the risk-neutral assumption of Black and Scholes[12], the expected investment return rate μ of underlying assets in eq. (21) is replaced by risk-free interest rate r, we see that the actuarial formula for the call option price turns out to be the well-known expression: The Black-Scholes option price formula in eq. (11) is obtained. The call option price derived in the actuarial method takes into account the expected return rate μ of the stock in terms of d_1 and d_2. The expected return rate μ is replaced by the risk-free interest rate r in the Black-Scholes formula.

3. Numerical Comparison between Black-Scholes Formula and Actuarial Model

This section provides some numerical results of simulation to illustrate how the two models determine the European call option price and to compare the results of the option prices obtained in the two models. Option pricing is performed on the basis of a series of daily closing stock price records from previous days. Using the past daily closing prices of the stock, we estimate the sample's expected investment return rate μ and volatility σ. We study the applicability of these models to the option prices in the market. In the numerical analysis, we use the daily closing stock prices of Construction Bank (China) in the Hong Kong Stock Market. Since future stock prices are not accurately predictable, we estimate the future expected return rate of the stock by the past values of the stock prices compiled in the following three ways: one past month data of the daily closing prices of November, two months data of October-November and three months data of September-November, 2010. Using the data of these stock prices in the past months, we evaluate the return rate and vola-

Table 3-1 The daily closing stock prices S and the least-square fitting process to obtain the return rate μ and volatility σ of Construction Bank, China in November, 2010.

	S	LnS	x	x^2	xy	$a+bx$	$\exp(a+bx)$	$dLnS$	sigma dz
2010/11/1	7.64	2.033398	0	0	0	2.093886	8.116394		
2010/11/2	7.81	2.055405	1	1	2.055405	2.086006	8.052688	0.022007	0.029887
2010/11/3	8.02	2.081938	2	4	4.163877	2.078126	7.989483	0.026533	0.034413
2010/11/4	8.27	2.112635	3	9	6.337904	2.070246	7.926773	0.030697	0.038577
2010/11/5	8.3	2.116256	4	16	8.465022	2.062366	7.864555	0.003621	0.011501
2010/11/8	8.12	2.09433	5	25	10.47165	2.054486	7.802826	−0.02193	−0.01405
2010/11/9	8.07	2.088153	6	36	12.52892	2.046606	7.741582	−0.00618	0.001703
2010/11/10	7.66	2.036012	7	49	14.25208	2.038726	7.680818	−0.05214	−0.04426
2010/11/11	7.72	2.043814	8	64	16.35051	2.030846	7.620531	0.007802	0.015682
2010/11/12	7.43	2.005526	9	81	18.04973	2.022966	7.560717	−0.03829	−0.03041
2010/11/15	7.34	1.993339	10	100	19.93339	2.015086	7.501372	−0.01219	−0.00431
2010/11/16	7.19	1.972691	11	121	21.6996	2.007206	7.442494	−0.02065	−0.01277
2010/11/17	7.11	1.961502	12	144	23.53803	1.999326	7.384078	−0.01119	−0.00331
2010/11/18	7.3	1.987874	13	169	25.84237	1.991446	7.32612	0.026372	0.034252
2010/11/19	7.23	1.978239	14	196	27.69535	1.983566	7.268617	−0.00963	−0.00175
2010/11/22	7.23	1.978239	15	225	29.67359	1.975686	7.211565	0	0.00788
2010/11/23	7.05	1.953028	16	256	31.24844	1.967806	7.154961	−0.02521	−0.01733
2010/11/24	7.06	1.954445	17	289	33.22557	1.959926	7.098802	0.001417	0.009297
2010/11/25	7.05	1.953028	18	324	35.1545	1.952046	7.043083	−0.00142	0.006463
2010/11/26	6.96	1.940179	19	361	36.86341	1.944166	6.987802	−0.01285	−0.00497
2010/11/29	7.08	1.957274	20	400	39.14548	1.936286	6.932954	0.017095	0.024975
2010/11/30	7.01	1.947338	21	441	40.89409	1.928406	6.878537	−0.00994	−0.00206
Σ	0.445503	44.24464	231	331	457.5889	0			

$b = 2.093886 \qquad a = \mu = -0.00788$

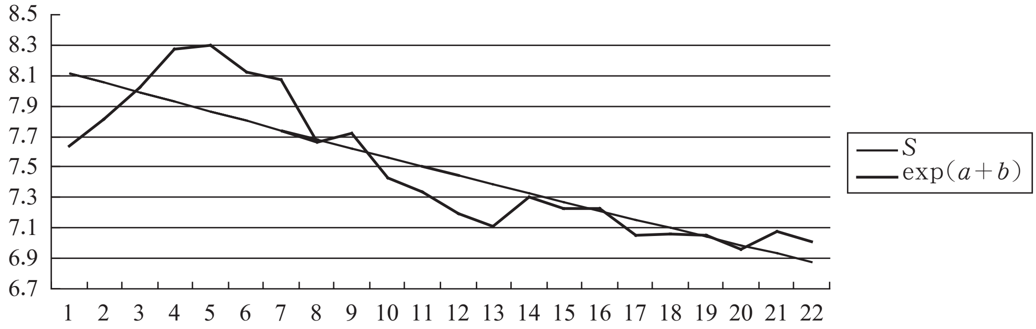

Fig. 3-1 The daily closing stock prices and the least-square fit line of the stock prices of Construction Bank, China in November, 2010. The return rate $\mu_1(=a)$ and volatility σ_1 are obtained from the least-square fitting.

tility of the stock price of the Construction Bank by the least-square fitting to the stock price data compiled in the three ways. The numerical results of the return rate and volatility obtained in the three ways of data compilation are $\mu_1 = -0.00788$, $\mu_2 = 0.00073$, $\mu_3 = 0.00233$, and $\sigma_1 = 2.09$, $\sigma_2 = 1.98$, $\sigma_3 = 1.89$. The least-square fitting process for the return rate and volatility is shown in Table 3-1 and the market stock prices in November, 2010 are shown in Fig. 3-1.

Because the numerical way to calculate μ_2 and μ_3 is the same as that of μ_1, only the Table and Fig. for μ_1 are shown in this paper.

1) *Search for a better setting for strike period of option*

In this subsection, the call option purchase date is set on December 1st, 2010. We calculate the option price in the actuarial model, using the return rate μ_1 of the stock prices for Construction Bank compiled during the month of November, 2010. The option exercise dates are chosen separately in three ways of $\tau = 30$, 60 and 90 days after the purchase date. The numerical results are shown in Table 3-2.

From Table 3-2, we see that, if the option exercise is 90 days after the purchase date, the standard deviation between the calculated option prices and the market prices results in the biggest value. Whereas, when the exercise is 30 days after the purchase, the standard deviation is the smallest. So, we conclude that, if the option exercise period τ becomes longer, the deviation between the theoretical values and the market values is larger.

2) *The search for a better setting of a data compilation period to evaluate the return rate and volatility*

In this subsection, we calculate the call option prices in the actuarial model, using the values for the expected return rate μ and volatility σ of Construction Bank, China, which were obtained by the stock price data compiled separately in three ways during one month, two months and three months. The option purchase date is set on December 1st, 2010. The exercise is done separately 30, 60 and 90 days after the purchase date. The numerical results are shown in Table 3-3. The standard deviations between the calculated option prices and the market prices are shown in Table 3-4. The values of return rate μ evaluated by the stock price compiled in the three ways are denoted as μ_1, μ_2 and μ_3 and the values of period τ for option to be exercised in the three ways are denoted as τ_1, τ_2 and τ_3.

In Tables 3-3 and 3-4, we see that the set of the return rate and volatility evaluated from the one-month data in November, 2010 yields the largest value of the standard deviation

Table 3-2 The prices calculated by using μ_1 and the market prices of the call option for Construction Bank, China. The standard deviations σ between the calculated prices and the market prices are also shown.

Exercised Prices	$\tau = 30$		$\tau = 60$		$\tau = 90$	
	Actuarial	Market	Actuarial	Market	Actuarial	Market
5	2.02676	2.03	2.023733	2.03		
5.25	1.776598	1.81	1.77342	1.81		
5.5	1.526436	1.56	1.523104	1.56		
5.75	1.276273	1.31	1.272712	1.31		
6	1.026097	1.07	1.021435	1.09	1.013648	1.11
6.25	0.775391	0.82	0.765766	0.86	0.752606	0.9
6.5	0.519722	0.59	0.503486	0.66	0.49125	0.71
6.75	0.260359	0.37	0.256243	0.46	0.257261	0.52
7	0.060441	0.23	0.07632	0.33	0.08867	0.39
7.25	−0.01157	0.13	−0.00606	0.22	0.00146	0.28
7.5	−0.00944	0.06	−0.01935	0.15	−0.02342	0.2
7.75	−0.00195	0.03	−0.01061	0.09	−0.01939	0.14
8	−0.00019	0.01	−0.00348	0.06	−0.01006	0.09
8.25	−1E-05	0.01	−0.0008	0.03		
8.5	−3.2E-07	0.01	−0.00014	0.02		
8.75	−6.1E-09	0.01	−1.8E-05	0.01		
σ	0.240456		0.493425		0.633728	

Table 3-3 Comparision table of the difference between the calculated option's prices and market prices

Exercise Price	$\tau = 30$				$\tau = 60$				$\tau = 90$			
	μ_1	μ_2	μ_3	Market Price	μ_1	μ_2	μ_3	Market Price	μ_1	μ_2	μ_3	Market Price
5	2.02676	2.0303	2.03096	2.03	2.023733	2.031	2.032	2.03				
5.25	1.776598	1.78031	1.781	1.81	1.77342	1.781	1.782	1.81				
5.5	1.526436	1.53033	1.53105	1.56	1.523104	1.531	1.532	1.56				
5.75	1.276273	1.28034	1.2811	1.31	1.272712	1.281	1.281	1.31				
6	1.026097	1.03036	1.03111	1.07	1.021435	1.03	1.031	1.09	1.013648	1.02928	1.02722	1.11
6.25	0.775391	0.78025	0.78029	0.82	0.765766	0.778	0.774	0.86	0.752606	0.77282	0.76696	0.9
6.5	0.519722	0.52744	0.52398	0.59	0.503486	0.518	0.513	0.66	0.49125	0.51152	0.50819	0.71
6.75	0.260359	0.2677	0.26632	0.37	0.256243	0.265	0.269	0.46	0.257261	0.26786	0.276	0.52
7	0.060441	0.05817	0.06719	0.23	0.07632	0.075	0.088	0.33	0.08867	0.08745	0.10359	0.39
7.25	−0.01157	−0.0123	−0.0115	0.13	−0.00606	−0.01	0	0.22	0.00146	−0.0028	0.00707	0.28
7.5	−0.00944	−0.0067	−0.0127	0.06	−0.01935	−0.02	−0.02	0.15	−0.02342	−0.0234	−0.0266	0.2
7.75	−0.00195	−0.0009	−0.0035	0.03	−0.01061	−0.01	−0.02	0.09	−0.01939	−0.0159	−0.0262	0.14
8	−0.00019	−4.00E-05	−0.0005	0.01	−0.00348	0	−0.01	0.06	−0.01006	−0.0066	−0.016	0.09
8.25	−1.00E-05	−1.00E-06	−4.00E-05	0.01	−0.0008	0	0	0.13				
8.5	−3.20E-07	−1.00E-08	−2.00E-06	0.01	−0.00014	0	0	0.02				
8.75	−6.10E-09	−9.00E-11	−7.00E-08	0.01	−1.80E-05	0	0	0.01				
σ	0.240456	0.27524	0.27213		0.493425	0.481	0.476		0.63372	0.61703	0.60956	

ps: σ is the standard deviation of the difference between the calculated price and market price

Table 3-4 The standard deviations between the calculated prices and the market prices

μ	$\tau = 30$	$\tau = 60$	$\tau = 90$
μ_1	0.281614	0.493425	0.633728
μ_2	0.275237	0.480834	0.617027
μ_3	0.27213	0.47618	0.60956

between the actuarial option prices and the market prices, while the set of the return rate and volatility obtained from the September–November three-month data in 2010 yields the smallest deviation. So, it can be concluded that a set of longer period data for stock return rate and volatility yields a smaller deviation between theory and market.

3) *Numerical comparison between the Black-Scholes formula and the Actuarial model*

In this subsection, we calculate the option prices in both the Actuarial and Black-Scholes models to compare the numerical results of the two models. We use the return rate and volatility of the stock prices of Construction Bank, China during one month of October, 2010 in Hong Kong Stock Market. The return rate is evaluated to be $\mu_4 = 0.012$. The option purchase date is set on November 1st, 2010, and the option is to be exercised 60 days after the purchase. The numerical results of the Black-Scholes model and the actuarial model are shown in Table 3–5.

In Table 3–5, we see that compared with the results of the Black-Scholes model, the standard deviation between theory and market in the Actuarial model is improved. In other words, the Actuarial model predicts more accurately the option prices in the market.

Table 3–5 The option prices calculated in the Actuarial and Black-Scholes models and the market prices. The standard deviations between the calculated prices and the market prices are also shown.

Exercise Prices	November–December		
	Actuarial	Black-Scholes	Market
3.4	4.242921	4.267831	4.24
3.6	4.043093	4.069468	4.04
3.8	3.843265	3.871105	3.84
4.0	3.643437	3.672742	3.64
4.2	3.443609	3.474379	3.44
4.4	3.24378	3.276016	3.24
4.6	3.043952	3.077653	3.04
4.8	2.844124	2.87929	2.84
4.9	2.74421	2.780109	2.74
5.0	2.644296	2.680927	2.64
5.25	2.394511	2.432974	2.39
5.5	2.144726	2.18502	2.14
5.75	1.89494	1.937067	1.89
6.0	1.645155	1.689113	1.64
6.25	1.39537	1.441159	1.39
6.5	1.145584	1.193206	1.14
6.75	0.895753	0.945246	0.9
7.0	0.644685	0.697028	0.68
7.25	0.386858	0.446208	0.46
7.5	0.143435	0.198297	0.3
7.75	0.005158	0.018617	0.18
8.0	−0.01438	−0.02914	0.1
8.25	−0.00407	−0.01283	0.05
8.5	−0.00044	−0.0021	0.03
8.75	−2.3E-05	−0.00016	0.02
9.0	−6.2E-07	−6.6E-06	0.01
σ	0.281818	0.295689	

4. Concluding Remarks

In this paper, we have examined the actuarial formula proposed by Bladt and Rydberg by using the actuarial option pricing concept. We quantitatively studied the option pricing constitution from the perspective of the assessment of potential losses and the corresponding probability distribution caused by implementing options, and then applied the option pricing model based on the Actuarial model, to the prices in market.

In detail, we have calculated the option prices for Construction Bank, China in the Actuarial model and the Black-Scholes model, and compared them with the prices in the market. Three discoveries were obtained: (1) If the option exercise is performed longer after the purchase, the deviation between the model value and the market value for the option price gets larger. (2) If the stock price data compiled during a longer term period are used to evaluate the stock return rate and volatility, the deviation gets smaller. (3) Compared with the numerical results of the option prices obtained by the Black-Scholes model, the Actuarial model gives the option prices deviated from the market prices with a smaller error. In other words, the Actuarial model can predict more accurately the option prices. Taking advantage of these discoveries, we may further improve the option pricing model. The applicability of the above three discoveries to other cases should be examined further in future studies.

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