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Optimized Schedule Allocation of Regular and Irregular Employees to Deal with Demand Quantity Fluctuation

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Abstract

We discuss the problem of whether the schedule allocation of only regular employees can efficiently deal with weekly fluctuating demand without the use of irregular employees. Conducting manufacturing operations seven days a week solves the problem. A new algorithm is presented to determine the minimal number of workers necessary for the fluctuating demand. The minimal numbers of workers are compared between the two Labor Allocation Models, i.e., Model a: Only regular employees are allocated, and Model b: Both regular and irregular employees are allocated. The present analysis shows that the schedule allocation of only regular employees can efficiently deal with fluctuating demand.

Key Words: Allocation of employees, Demand quantity fluctuation, Regular employment

1. Introduction

Demand quantity for products fluctuates seasonally, weekly and by the statistical outcome of consumers' arbitrary purchasing. The employment and scheduling of irregular employees is an inexpensive system to deal with fluctuating demand. Actually, enterprises, pursuing fierce competition on a global scale, allocate irregular employees in an aim to reduce the cost due to the superfluous hiring of regular employees because of fluctuating demand [1]. As a result, presently in Japan, the number of irregular employees has risen up to 40% of the labor force. Irregular employment is giving rise to social differentials, and discouraging young people from getting married, among other things. The hiring and scheduling of regular employees is a proper employment in enterprises. On this note, as for the problem of whether the schedule allocation of only regular employees can efficiently deal with fluctuating demand, we will show that the allocation of only regular employees does not necessitate a superfluous number of workers compared with the use of irregular employees as filling in the demand fluctuation. For the purpose of this study, we define a regular employee as full-time employee working five days a week. An irregular employee is defined as part-time employee.

The scheduling of workers has been studied to deal with fluctuating demand quantity in the literature. The algorithms have been developed substantially for the two cases, i.e., for (1) a restricted set of demand requirements and (2) the general cases of arbitrary cyclic demands. Baker and Magazine developed the work force scheduling for the restricted case where demand requirements are constant across weekdays and constant across weekends. They gave a formula for the minimal work force size and a special scheduling algorithm to allocate the work force such that the demand requirements are met [2]. As for the case of scheduling for arbitrary cyclic demands, Bechtold presented an iteration method to allocate the minimal number of full-time workers on a seven-day week demand cycle, while allowing the workers to have two consecutive days off per cycle. Their method has some arbitrariness in allocating workers in the iteration procedure. Hence, the minimal number of workers is not determined until the end of the iteration procedure [3]. Burns and Carter determined the work force size, imposing three constraints on the lower bound, i.e., weekend constraint, total demand constraint and maximum daily demand constraint. After the determination of the work force size, they go to the second procedure to schedule the work force for operation [4]. Now, various types of software are available such that work force scheduling is worked out even under complicated scheduling constraints. Any of these algorithms do not have a universal framework to determine the minimal number of workers and the schedules simultaneously which can be implemented on a manual basis for any arbitrary cyclic demand. The development of algorithms is necessary to invent a new technology for future software.

In the present paper, we propose a new algorithm to determine the minimal number of employees to deal with the weekly fluctuating demand quantity. The advantage of the present algorithm is to have a general framework of scheduling to determine the minimal number of employees and their schedules for any arbitrary cyclic demand. This algorithm, which is based on mathematical integer linear programming, is applied to the comparison between the minimal numbers of employees required in the two Labor Allocation Models, i.e., Model a: the hiring of only regular employees, and Model b: the hiring of both regular and irregular employees. The present algorithm formulates a framework to determine the minimal numbers of employees symmetrically in the two Models. This algorithm can be implemented on a manual basis. Hence, we determine generally the minimal number of employees for any arbitrary cyclic demand.

2. Schedule Allocation of Employees optimized to Deal with Fluctuating Demand

In order to discuss the problem of whether the schedule allocation of only regular employees can efficiently deal with fluctuating demand, we will determine the minimal numbers of employees necessary for the work operation such that the following constraints are satisfied:

a) The enterprise operates their manufacturing activity for 7 days a week, including the weekend.

b) An example of the number of workers required for working on any given day of the week is given in Table 1. The uneven distribution of the required number of employees on days of the week is a mathematical model standing for weekly fluctuation of demand quantity. The present algorithm determines the optimal numbers of workers for any other cases of arbitrary cyclic demands, as will be shown later.

c) The regular employees work for five consecutive days a week.

d) The regular employees are divided into 7 groups. Each a group starts working on a given day of the week.

e) The enterprise may employ irregular workers to work on the days which the enterprise decides of their own accord.

Table 1 The number of employees required for working on any given day of the week

M	Tu	W	Th	F	Sa	Su
17	13	15	19	14	16	11

We will compare the necessary numbers of workers between the following two Labor Allocation Models in order to study the efficiency of regular employment to deal with the fluctuating demand:

Labor Allocation Model a: Only regular employees are allocated to satisfy the number of workers required on any given day of the week shown in Table 1. The number of employees should be minimized.

Labor Allocation Model b: Regular and irregular employees are allocated to meet the number of workers required on any given day of the week in Table 1. The number of irregular employees is minimized.

In Subsections 1) and 2), we will determine the number of employees necessary in Labor Allocation Models a and b, respectively.

1) *Labor Allocation Model a*

In the Model a [5], more employees than the number required on a day of the week in Table 1 should be supplied by five groups of regular employees. Hence, the following coupled inequalities are set up:

$$\begin{aligned}
x_1 \qquad\qquad\qquad +x_4+x_5+x_6+x_7 &\geqq 17,\\
x_1+x_2 \qquad\qquad\qquad +x_5+x_6+x_7 &\geqq 13,\\
x_1+x_2+x_3 \qquad\qquad\qquad +x_6+x_7 &\geqq 15,\\
x_1+x_2+x_3+x_4 \qquad\qquad\qquad +x_7 &\geqq 19,\\
x_1+x_2+x_3+x_4+x_5 \qquad\qquad\qquad &\geqq 14,\\
x_2+x_3+x_4+x_5+x_6 \qquad\qquad &\geqq 16,\\
x_3+x_4+x_5+x_6+x_7 &\geqq 11,
\end{aligned} \tag{1}$$

where a non-negative integer x_i stands for the number of regular employees in the group who start working on day i of the week. Ordering the days of the week in the order starting with Monday, we express Monday as $i = 1$, Tuesday as $i = 2$ and so on.

Using the mathematical integer linear programming method, we will minimize the total number of the regular employees,

$$z = x_1+x_2+x_3+x_4+x_5+x_6+x_7,$$

subject to the inequality condition (1) expressed in terms of non-negative integers x_i.

As a first step for the optimization, supposing that the number of employees required on a day of the week in Table 1 is met by supplying five groups of regular employees, we solve the following coupled equations to estimate the number of employees in each group:

$$
\begin{aligned}
x_1 \qquad\qquad\quad + x_4 + x_5 + x_6 + x_7 &= a_1 = 17,\\
x_1 + x_2 \qquad\qquad\quad + x_5 + x_6 + x_7 &= a_2 = 13,\\
x_1 + x_2 + x_3 \qquad\qquad\quad + x_6 + x_7 &= a_3 = 15,\\
x_1 + x_2 + x_3 + x_4 \qquad\qquad\quad + x_7 &= a_4 = 19,\\
x_1 + x_2 + x_3 + x_4 + x_5 \qquad\qquad\quad &= a_5 = 14,\\
x_2 + x_3 + x_4 + x_5 + x_6 \qquad\quad &= a_6 = 16,\\
x_3 + x_4 + x_5 + x_6 + x_7 &= a_7 = 11,
\end{aligned}
\tag{2}
$$

where a_j is the required number of workers on day j of the week in Table 1. The solution of the above coupled equations is

$$
\begin{aligned}
x_1 &= 3/5\,(a_1 + a_3 + a_5) - 2/5\,(a_7 + a_2 + a_4 + a_6) = 4,\\
x_2 &= 3/5\,(a_2 + a_4 + a_6) - 2/5\,(a_1 + a_3 + a_5 + a_7) = 6,\\
x_3 &= 3/5\,(a_3 + a_5 + a_7) - 2/5\,(a_2 + a_4 + a_6 + a_1) = -2,\\
x_4 &= 3/5\,(a_4 + a_6 + a_1) - 2/5\,(a_3 + a_5 + a_7 + a_2) = 10,\\
x_5 &= 3/5\,(a_5 + a_7 + a_2) - 2/5\,(a_4 + a_6 + a_1 + a_3) = -4,\\
x_6 &= 3/5\,(a_6 + a_1 + a_3) - 2/5\,(a_5 + a_7 + a_2 + a_4) = 6,\\
x_7 &= 3/5\,(a_7 + a_2 + a_4) - 2/5\,(a_6 + a_1 + a_3 + a_5) = 1,
\end{aligned}
\tag{3}
$$

The estimated numbers $x_3 = -2$ and $x_5 = -4$ in the above solution are not acceptable for the workers in groups $i = 3$ and 5. The lowest component $x_5 = -4$ of the solution (3) is denoted as the most singular. The singularity of the estimated x_5 is originated from the smallest triplet $a_5 = 14$, $a_7 = 11$ and $a_2 = 13$ of the numbers of employees required for working on every other day of the week in Table 1. In any general case of arbitrary cyclic demands, the lowest of x_i's (such as x_5 in the present case) in the solution (3) of the coupled equations determines the optimum number of employees as will be shown later.

As for the coupled inequalities (1), assigning c_j as the number of supplementary (slack) employees on day j of the week, we replace the coupled inequalities by the following coupled equations:

$$
\begin{aligned}
x_1 \qquad\qquad\quad + x_4 + x_5 + x_6 + x_7 &= 17 + c_1,\\
x_1 + x_2 \qquad\qquad\quad + x_5 + x_6 + x_7 &= 13 + c_2,\\
x_1 + x_2 + x_3 \qquad\qquad\quad + x_6 + x_7 &= 15 + c_3,\\
x_1 + x_2 + x_3 + x_4 \qquad\qquad\quad + x_7 &= 19 + c_4,\\
x_1 + x_2 + x_3 + x_4 + x_5 \qquad\qquad\quad &= 14 + c_5,\\
x_2 + x_3 + x_4 + x_5 + x_6 \qquad\quad &= 16 + c_6,\\
x_3 + x_4 + x_5 + x_6 + x_7 &= 11 + c_7,
\end{aligned}
\tag{4}
$$

where the number c_j of supplementary workers should be non-negative integers. In terms of the numbers x_i of the workers in group i, the sum of c_j's is expressed as

$$
\begin{aligned}
& c_1 + c_2 + c_3 + c_4 + c_5 + c_6 + c_7\\
&= 5\,(x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 - 21)\\
&= 5n > 0
\end{aligned}
\tag{5}
$$

with a positive integer n.

Now we start to minimize the number of the employees,

$$z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7,$$

i.e., the sum of c_j's, subject to the condition of equations (4) expressed in terms of non-negative integers x_i. The solution of the coupled equations (4) is

$$\begin{aligned}
x_1 &= 4+3/5\,(c_1+c_3+c_5)-2/5\,(c_7+c_2+c_4+c_6),\\
x_2 &= 6+3/5\,(c_2+c_4+c_6)-2/5\,(c_1+c_3+c_5+c_7),\\
x_3 &= -2+3/5\,(c_3+c_5+c_7)-2/5\,(c_2+c_4+c_6+c_1),\\
x_4 &= 10+3/5\,(c_4+c_6+c_1)-2/5\,(c_3+c_5+c_7+c_2),\\
x_5 &= -4+3/5\,(c_5+c_7+c_2)-2/5\,(c_4+c_6+c_1+c_3),\\
x_6 &= 6+3/5\,(c_6+c_1+c_3)-2/5\,(c_5+c_7+c_2+c_4),\\
x_7 &= 1+3/5\,(c_7+c_2+c_4)-2/5\,(c_6+c_1+c_3+c_5),
\end{aligned} \tag{6}$$

where the supplementary terms c_j are necessary to make the component x_5 non-negative that was the most singular in the solution (3). The non-negative integer constraint of x_5 restricts the values of the c_j's so that

$$x_5 = -4+3/5\,(c_5+c_7+c_2)-2/5\,(c_4+c_6+c_1+c_3) \geqq 0, \tag{7}$$

which requires

$$c_5+c_7+c_2 \geqq 4\times 5/3+2/3\,(c_4+c_6+c_1+c_3) \geqq 4\times 5/3.$$

Hence,

$$c_1+c_2+c_3+c_4+c_5+c_6+c_7 \geqq 4\times 5/3 = 6.66. \tag{8}$$

Owing to the relationship of (5) and (8), the optimization requirement that the number of employees is minimized is satisfied by setting the sum of c_j's

$$c_1+c_2+c_3+c_4+c_5+c_6+c_7 = 5n = 10 \tag{9}$$

with $n = 2$. Therefore, Eq. (5) leads to the minimal number of regular employees

$$\begin{aligned}
&x_1+x_2+x_3+x_4+x_5+x_6+x_7\\
&= 21+1/5\,(c_1+c_2+c_3+c_4+c_5+c_6+c_7)\\
&= 21+n = 21+2 = 23.
\end{aligned}$$

These results indicate that the sum of the required numbers of workers on 7 days of the week in Table 1 is 105. Since regular employees work on 5 days a week, regular workers of $105/5 = 21$ are required at the least. In addition to the 21 workers, however, Eq. (9) shows that supplementary $n = 2$ workers are necessary. Therefore, totally 23 regular employees are necessary.

The non-negative integers x_3, x_5 and x_7 in the solution (6) of the coupled equations restrict the values of the supplementary variables c_j. The non-negative integer,

$$\begin{aligned}
x_3 &= -2+3/5\,(c_3+c_5+c_7)-2/5\,(c_2+c_4+c_6+c_1)\\
&= -2+3\times 10/5-(c_2+c_4+c_6+c_1) \geq 0,
\end{aligned}$$

which requires

$$c_2+c_4+c_6+c_1 \leq 4. \tag{10}$$

Similarly, $x_5 \geq 0$ and $x_7 \geq 0$ require respectively

$$c_4+c_6+c_1+c_3 \leq 2, \tag{11}$$

and

$$c_6 + c_1 + c_3 + c_5 \leq 7. \tag{12}$$

The combination of constraints (9)-(12) yields the redundant solutions of the coupled equations (4). One example of the optimized schedules of 23 regular employees is shown in Table 2.

Table 2 A. The number x_i of regular employees in group i, and B. The number of employees working on a day of the week in Labor Allocation Model a are shown.

A

Group	1	2	3	4	5	6	7
No. Regular Employees	3	5	1	6	1	3	4

B

Day of Week	M	Tu	W	Th	F	Sa	Su
No. Regular Employees	17	16	16	19	16	16	15

The present algorithm to determine the minimal number of regular employees is applicable to any arbitrary cyclic demand fluctuation. If a cyclic demand fluctuation yields a lowest component $x_i = -s$ in the solution (3), the minimal number of regular employees is determined to be the upper integer part of $N/5 + s/3$, where N is the total number of employees required on seven days a week for the cyclic demands.

2) *Labor Allocation Model b*

In the Model b, the sum of the numbers of regular and irregular employees is minimized. Since irregular employees are hired by the enterprise decision to deal with the demand fluctuation, the optimized number of employees working on a day of the week can be equal to the required number of employees in Table 1. Therefore, we set up the following coupled equations for the numbers of regular and irregular employees required on a day of the week:

$$\begin{aligned}
x_1 \qquad\qquad\quad + x_4 + x_5 + x_6 + x_7 + b_1 &= 17,\\
x_1 + x_2 \qquad\qquad\quad + x_5 + x_6 + x_7 + b_2 &= 13,\\
x_1 + x_2 + x_3 \qquad\qquad\quad + x_6 + x_7 + b_3 &= 15,\\
x_1 + x_2 + x_3 + x_4 \qquad\qquad\quad + x_7 + b_4 &= 19,\\
x_1 + x_2 + x_3 + x_4 + x_5 \qquad\qquad\quad + b_5 &= 14,\\
x_2 + x_3 + x_4 + x_5 + x_6 \qquad + b_6 &= 16,\\
x_3 + x_4 + x_5 + x_6 + x_7 + b_7 &= 11,
\end{aligned} \tag{13}$$

where b_j stands for the number of irregular employees working on day j of the week. In terms of the number x_i of the regular employees in group i, the sum of b_j's is expressed as

$$\begin{aligned}
& b_1 + b_2 + b_3 + b_4 + b_5 + b_6 + b_7\\
= & -5\,(x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 - 21)\\
= & \ 5n > 0
\end{aligned} \tag{14}$$

with a positive integer n.

Using the mathematical integer linear programming method, we will minimize the number of irregular employees,

$$z = b_1 + b_2 + b_3 + b_4 + b_5 + b_6 + b_7,$$

subject to the condition of the coupled equations (13) expressed in terms of non-negative integers x_i and b_j. The optimization of z is carried out in a way symmetric to that in Subsection 1).

The solution of the coupled equations (13) is

$$\begin{aligned}
x_1 &= 4+2/5\,(b_7+b_2+b_4+b_6)-3/5\,(b_1+b_3+b_5),\\
x_2 &= 6+2/5\,(b_1+b_3+b_5+b_7)-3/5\,(b_2+b_4+b_6),\\
x_3 &= -2+2/5\,(b_2+b_4+b_6+b_1)-3/5\,(b_3+b_5+b_7),\\
x_4 &= 10+2/5\,(b_3+b_5+b_7+b_2)-3/5\,(b_4+b_6+b_1),\\
x_5 &= -4+2/5\,(b_4+b_6+b_1+b_3)-3/5\,(b_5+b_7+b_2),\\
x_6 &= 6+2/5\,(b_5+b_7+b_2+b_4)-3/5\,(b_6+b_1+b_3),\\
x_7 &= 1+2/5\,(b_6+b_1+b_3+b_5)-3/5\,(b_7+b_2+b_4),
\end{aligned} \tag{15}$$

where the component x_5 that was the lowest in the solution (3) should be non-negative. Therefore,

$$x_5 = -4+2/5\,(b_4+b_6+b_1+b_3)-3/5\,(b_5+b_7+b_2) \geqq 0, \tag{16}$$

which requires

$$b_4+b_6+b_1+b_3 \geqq 4\times 5/2+3/2\,(b_5+b_7+b_2) \geqq 4\times 5/2.$$

Hence,

$$b_1+b_2+b_3+b_4+b_5+b_6+b_7 \geqq 4\times 5/2 = 10. \tag{17}$$

Owing to the relationship of (14) and (17), the number of irregular employees is minimized by setting the sum of b_j's,

$$b_1+b_2+b_3+b_4+b_5+b_6+b_7 = 5n = 10 \tag{18}$$

with $n = 2$. Therefore, Eq. (14) leads to the optimum number of regular employees,

$$\begin{aligned}
&x_1+x_2+x_3+x_4+x_5+x_6+x_7\\
&= 21-1/5\,(b_1+b_2+b_3+b_4+b_5+b_6+b_7)\\
&= 21-n = 21-2 = 19.
\end{aligned}$$

These results indicate that 10 irregular employee-days and 19 regular employees are necessary.

The non-negative integers x_1, x_3, x_5 and x_7 in the solution (15) of the coupled equations restrict the values of the supplementary variables b_j. The non-negative integer,

$$\begin{aligned}
x_1 &= 4+2/5\,(b_7+b_2+b_4+b_6)-3/5\,(b_1+b_3+b_5)\\
&= 4+2\times 10/5-(b_1+b_3+b_5) \geq 0,
\end{aligned}$$

which requires

$$b_1+b_3+b_5 \leq 8. \tag{19}$$

Similarly, $x_3 \geq 0$, $x_5 \geq 0$ and $x_7 \geq 0$ require respectively

$$b_3+b_5+b_7 \leq 2, \tag{20}$$

$$b_5+b_7+b_2 \leq 0, \tag{21}$$

and

$$b_7 + b_2 + b_4 \leq 5. \tag{22}$$

The combination of constraints (18)-(22) yields the redundant solutions b_j's for the optimized number of workers. We obtained an optimum solution that 3 irregular employees are necessary in addition to 19 regular employees. 2 irregular employees work for 3 days a week and another 1 works for 4 days. One example of the optimum numbers of the regular employees in groups, and of the regular and irregular employees working on a day of the week are shown in Table 3.

Table 3 A. The number of regular employees in a group, and B. The numbers of regular and irregular employees working on a day of the week in Labor Allocation Model b are shown.

A

Group	1	2	3	4	5	6	7
No. Regular Employees	4	4	1	5	0	3	2

B

Day of Week	M	Tu	W	Th	F	Sa	Su
No. Regular Employees	14	13	14	16	14	13	11
No. Irregular Employees	3	0	1	3	0	3	0

We have seen that Labor Allocation Model a requires 23 employees and Model b 22 employees. The allocation of only regular employees does not necessitate a superfluous number of workers compared with the case of using irregular employees.

The present algorithm is applicable to determine the minimal value of employee-days of irregular employees for any arbitrary cyclic demand fluctuation. If the lowest component of the solution (3) of the coupled equations is $x_i = -s$, the minimal value of irregular employee-days is determined to be $N-5\,[N/5-s/2]$, where N is the total number of employees required on seven days a week for the cyclic demands and $[x]$ is the lower integer part of x. The optimum number of regular employees is $[N/5-s/2]$.

3. Concluding Remarks

We have presented a new algorithm to determine the minimal number of employees necessary to deal with the fluctuation of demand quantity. By optimizing the numbers of employees necessary in the Labor Allocation Models a and b, we have found that the schedule allocation of only regular employees does not necessitate a superfluous number of workers compared with the case of using irregular employees as filling in the demand fluctuation. The present algorithm can be applied to general cases of demand fluctuation, whatever the fluctuation cycle is. Hence, it is applicable to the cases of demand quantity fluctuating seasonally. In the application to any case, the reader will obtain the optimized number of employees from Eq.'s (8) and (17). The general structure of the present algorithm shows that the minimal numbers of employees are almost equal to each other between the two Labor Allocation Models. This algorithm may be applicable to non-cyclic demand fluctuation. Even in the extended cases, the enterprise can allocate employees by predicting the fluctuation pattern of demand quantity in each a period [6].

It is concluded that regular employment can deal with fluctuating demand as efficiently as irregular employment. This conclusion may suggest a hint for enterprise managers to take a step to promote the proper employment, i.e., the allocation of regular employees, which would get rid of the social differentials and encourage young people to get married without fearing unstable employment.

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