Growth and Disequilibrium Pricing

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I. Introduction

Around the relevance of Marx's economics as a foundation for a deeper insight of the capitalist economy, many authors have produced either supporting or rejecting writings over decades. There is no need to mention here that the core of the argument lies in the appraisal of Marx's analysis of value and surplus value. It has been pointed out more often than not that Marx's theory of value will come up against difficulties if joint-production, choice of techniques and heterogeneous labour are explicitly considered. One can find many contributors on these difficulties such as Morishima, Steedman and etc.

Nevertheless, the relevance of a theory should not be judged by mere existence of difficulties. As Newton physics can still hold its relevance after the appearance of the relativity principle, so can do Marx's economics. What is necessary is to develope the theory up to the boundary of its possibility.

Marx's theory of value and surplus value holds consitently, even if one does not confine oneself to stating problems in terms of the two departmental economy as in Das Kapital II. The two departmental economy á la Marx, however, is still very instructive for arguing basic problems in economics. It was shown in Fujimori (1978) that Marx's theory of value was helpful for understanding antagonistic relations between the two major classes. That is, uneven growth was seen to have much to do with class conflicts, if viewed from the standpoint of value analysis.

The aim of this short paper is to make some direct supplementaries to Fujimori (1978), for various points were left untouched therein. In the outset, it was supposed that the equilibrium price system held throughout the periods. However, some explanations would be required to justify this, because in the discussion of uneven growth, which is a part of disequilibrium analysis, it may not be plausible to assume that equilibrium pricing takes place momentarily. There may exist a profit-rate difference between departments from time to time. The main point to be tackled in this paper is what kind of effect pricing with the profit-rate difference, which will be called disequilibrium pricing, exerts on uneven growth.

In section II, disequilibrium price regions will be defined. Also one will get the profit-rates curve, and the relation between disequilibrium price regions and the profit-rate curve will be made clear.

In section III, relations between disequilibrium pricing and the transformation will be discussed. Though the proposition obtained in this section has rather an eccentric atmospher, it will not be difficult to construe its implications.

The framework of the analysis hereafter follows that of Fujimori (1978). For the sake of convenience, however, principal symbols and necessary assumptions will be repeated below, which will make this paper as independent as possible.

The discussion will be described by the usual Marxian two departmental economy. The department I (abbreviated to dept. I) supplies the means of production or capital goods, whilst the department II (also to dept. II) produces consumption goods.

The economy is assumed to have two major social classes—the capitalist class and the working class. Capitalists obtain profit, whilst workers receive wages. Workers are supposed to do no saving.

It is assumed that techniques are linear. Each kind of goods is reproduced in a unit period, in whose production process point-input and point-output take place. Moreover. choice of techniques and fixed capital are excluded. The economy is presupposed to be able to produce surplus product.

Major symbols employed for the description of the subsequent argument are:

a_i : input coefficient

L_i : labour coefficient

b : real wages

x_i : quantity

x_i^a : actual quantity

p_i : price

p^r : $=p_1/p_2$

w_i : value

g_i : growth-rate

π_i : profit-rate

γ : rate of capitalist consumption (=capitalist consumption/x_2)

In vector or matrix notation, one has

$$A=\begin{pmatrix} a_1 & a_2 \\ 0 & 0 \end{pmatrix}, \quad L=(L_1, \ L_2), \quad f=\begin{pmatrix} 0 \\ b \end{pmatrix}, \quad M=A+fL=\begin{pmatrix} a_1 & a_2 \\ bL_1 & bL_2 \end{pmatrix}$$

$$x=\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \quad w=(w_1, \ w_2), \quad p=(p_1, \ p_2).$$

Subscripts $i=1$ or 2 indicate the department concerned or goods produced by the department. Without loss of generality, one may put

$$L_1=L_2=L_c$$

by taking an appropriate unit in each department.

With these notations given, one can paraphrase mathematically the fact that the economy can produce surplus products: there exists $x>0$ such that $x>Mx$. This can be reduced to

$$1-a_1>0$$

and

$$1-bL_c(1+a_2/(1-a_1))>0$$

in the two departmental case.

Needless to say, technical coefficients and real wages are assumed to be non-negative.

II. Disequilibrium price regions

Let us begin with the price system having no profit-rate difference between departments, which is called the equilibrium price system. Though the purpose of this paper is to deal with disequilibrium pricing, the longrun equilibrium price will be found to be still a core of pricing.

The equilibrium price system is described by

$$p_1=(1+\pi)(p_1a_1+p_2bL_c),$$

$$p_2=(1+\pi)(p_1a_2+p_2bL_c),$$

or, in shorthand, by

$$p=(1+\pi)pM. \qquad \text{(II-1)}$$

Let P^E denote the set of equilibrium prices:

$$P^E=\{p|\pi>0,\ \ p>0 \text{ in (II-1)}\}$$

Since the surplus condition is fulfilled, one has $P^E \neq \emptyset$, which is represented by a half-line in R^2_+ starting from the origin. Needless to say, π is determined by

$$\pi=1/\lambda_M-1, \qquad \text{(II-2)}$$

where λ_M is the Frobenius-root of M, and p by the eigen-vector associated with it.

The two findings about the equilibrium price should be mentioned here. In the first place, the relationship between π and b is illustrated by the wage-profit curve, The major property of the curve is expressed by the formula:

$$\partial\pi/\partial b<0. \qquad \text{(II-3)}$$

This shows, as well-known, that wages and profit are in an antagonistic relation.

In the second place, a simple mathematical manipulation yields the formula:

$$\partial p^r/\partial \pi > 0. \qquad \text{(II-4)}$$

This formula shows that the relative price of capital goods is an increasing function of the equilibrium profit-rate. This naturally means that p_2/p_1 is an increasing function of b. This fact is rather important for the discussion of disequilibrium pricing.

Now, let us suppose that the price system is in disequilibrium with different profit-rates: one has

$$p_1 = (1+\pi_1)(p_1 a_1 + p_2 b L_c),$$

$$p_2 = (1+\pi_2)(p_1 a_2 + p_2 b L_c),$$

or, in vector-matrix notation,

$$p = pM(I+\hat{\pi}), \qquad \text{(II-5)}$$

where $\hat{\pi}$ is the diagonal matrix made of π_is. Let P^{DE} denote the set of all possible prices:

$$P^{DE} = \{p | p > 0,\ \pi_1 > 0,\ \pi_2 > 0 \text{ in (II-5).}\}$$

One can also get by simple calculation that $p \in P^{DE}$ is equivalent to

$$bL_c/(1-a_1) < p^r < (1-bL_c)/a_2.$$

It is trivial that $P^E \subseteq P^{DE}$. So, $P^E \neq \emptyset$ implies $P^{DE} \neq \emptyset$. Moreover, it is seen that in this case $P^{DE} \neq \emptyset$ implies $P^E \neq \emptyset$. That is:

Proposition 1.

$$P^E \neq \emptyset \Leftrightarrow P^{DE} \neq \emptyset.$$

Proof.

One has only to prove "$\Leftarrow$" part. In this case, $P^{DE} \neq \emptyset$ is equivalent to

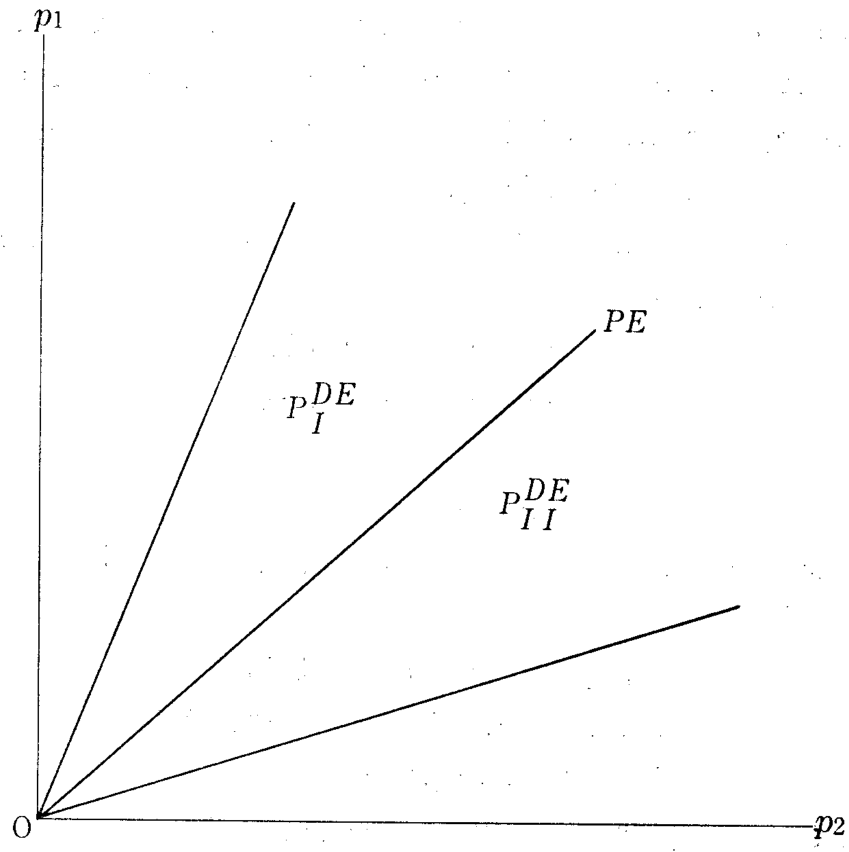
$$0 < bL_c/(1-a_1) < (1-bL_c)/a_2.$$

Since a_i, b and L_c are all positive, these inequalities are reduced to the surplus condition. Q. E. D.

Thus, one can illustrate P^E and P^{DE} as in the following Figure 1. By the half-line representing P^E, P^{DE} is split into two parts, which will be denoted as P_{I}^{DE} and P_{II}^{DE} clockwise.

Let us turn to the relationship between π_1 and π_2. Since π_1 is not necessarily equal to π_2, one has to find how π_1 moves in responce to changes in π_2. By eliminating p_1 and p_2 from (II-5), one gets, if $a_1 \neq a_2$, the function

Figure 1.



$$\pi_1 + k_1 = k_2/(k_3\pi_2 + k_4) \qquad \text{(II-6)}$$

where k_is are temporary parameters:

$$k_1 = (1 - a_1 + a_2)/(a_2 - a_1),$$
$$k_2 = a_2/(a_2 - a_1),$$
$$k_3 = (a_2 - a_1)bL_c,$$

and

$$k_4 = a_1(1 - bL_c) + a_2 bL_c.$$

Bearing in mind that obviously $1 - a_1 + a_2 > 0$ and $k_4 > 0$, one has $k_1 \gtrless 0$, $k_2 \gtrless 0$ and $k_3 \gtrless 0$ according as $a_2 \gtrless a_1$ respectively. If $a_1 = a_2$, the function becomes linear as

$$\pi_1 = -(bL_c/a_1)\pi_2 + (1 - a - bL_c).$$

The positivity of the constant in the right-hand side is endorsed by the surplus condition. For, the economy can produce surplus products if and only if $1 > a_1 + bL_c$.

As for the maximum of π_1 and π_2, from (II-5) one gets

$$p^r = bL_c/(1/(1 + \pi_1) - a_1)$$

and,

$$p^r = (1/(1+\pi_2) - bL_c)/a_2,$$

and so, $p^r > 0$ if and only if

$$\pi_1 \leqq (1/a_1) - 1,$$

and

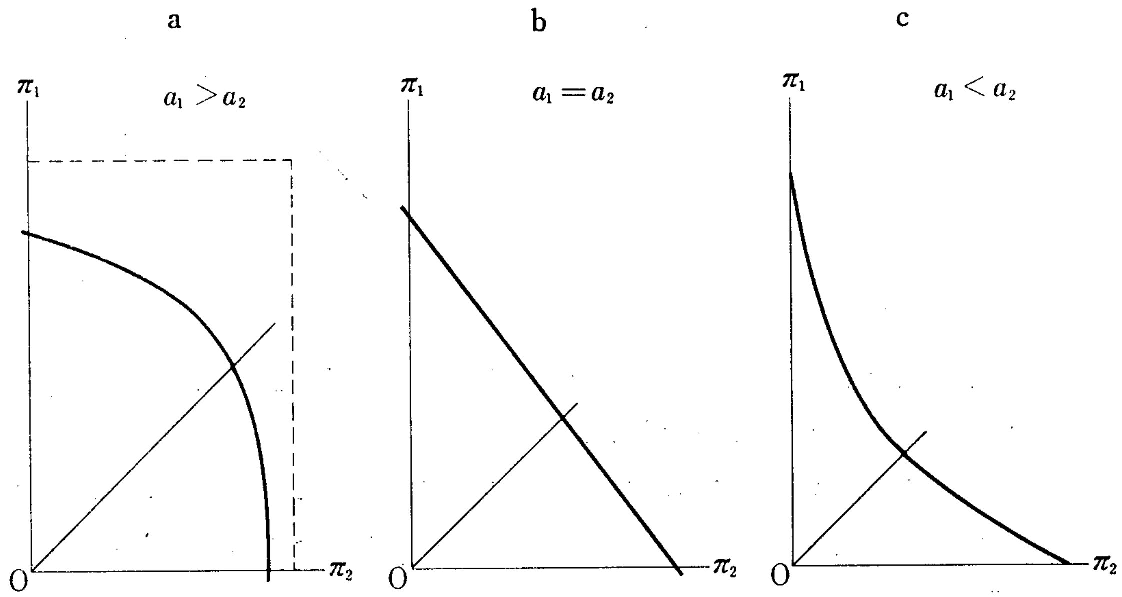
$$\pi_2 \leqq (1/bL_c) - 1.$$

Whether a_1 is greater or smaller than, or equal to a_2, one soon gets the formula:

$$\partial\pi_1/\partial\pi_2 < 0.$$

The following figures—Figure 2 a–c—illustrate the relationship between π_1 and π_2, the profit rate curve, in three cases with respect to a_1 and a_2.

Figure 2.



Using the function deduced as above, one can point out some other formulae:

$$\partial p^r/\partial\pi_1 > 0$$

and evidently,

$$\partial p^r/\partial\pi_2 < 0.$$

By taking into account all these formulae, let us try to synthesize the relationship of p_1 and p_2, and that of π_1 and π_2. Suppose that weighted sum of p_1 and p_2 is fixed as

$$p_1z_1+p_2z_2=K. \tag{II-7}$$

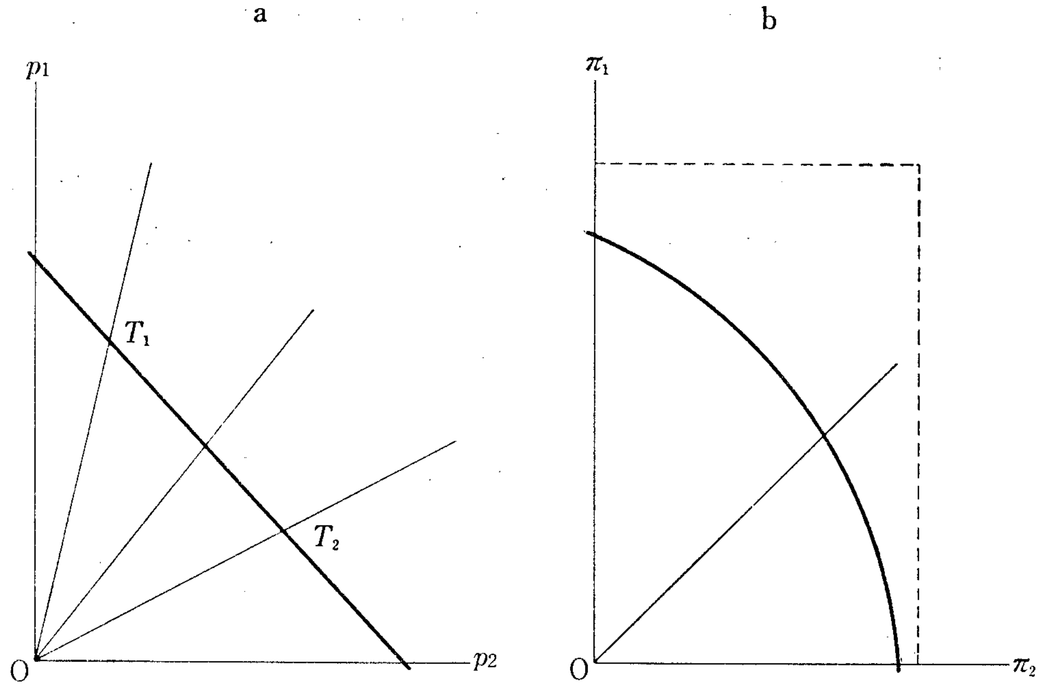
Let us define two sets:

$$P^{DE}_{(K)}=\{p|\pi_1>0,\ \pi_2>0,\ p>0 \text{ in (II-5) and (II-7)}\}$$

and

$$\Pi=\{(\pi_1,\ \pi_2)|\pi_1>0,\ \pi_2>0 \text{ in (II-6)}\}.$$

Figure 3.



$P^{DE}_{(K)}$ is illustrated by the line-segment T_1T_2 in Figure 3 a. Whilst, Figure 3 b shows the $\pi_1-\pi_2$ curve with $a_1>a_2$.

Now, one can say that p moves on T_1T_2 from left to right as $(\pi_1,\ \pi_2)$ moves from left to right on the profit rate curve, and vice-versa. Then, it can be said that there exists homeomorphism between $P^{DE}_{(K)}$ and Π, because the mapping $P^{DE}_{(K)}\rightarrow\Pi$ and its inverse are continuous, and because the closures of them are both compact. That is, one gets:

Proposition 2.

Π is homeomorhic to $P^{DE}_{(K)}$.

III. Disequilibrium pricing and the transformation theorems

Suppose that the economy is in the state of extended reproduction. The actual quantity system of the economy will be expressed by

$$x_1^a = (1+g_1)a_1x_1^a + (1+g_2)a_2x_2^a$$
$$(1-\gamma)x_2^a = bL_c\{(1+g_1)x_1^a + (1+g_2)x_2^a\}$$

or, in a brief manner,

$$x^a = M(\gamma)(I+\hat{g})x^a, \qquad \text{(III-1)}$$

where $M(\gamma) = I(\gamma)^{-1}M$, $I(\gamma) = \begin{pmatrix} 1 & 0 \\ 0 & 1-\gamma \end{pmatrix}$ and $\hat{g}$ is the diagonal matrix of g_is.

Now, the value system which is hidden behind the price system can be expressed by

$$w = wA + L = L(I-A)^{-1}. \qquad \text{(III-2)}$$

The first principle of Marx's transformation asserts that the total price of commodities produced is equal to their total value. This implies that the total volume of commodities is independent of conditions in the market, because values of commodities are free from the market once choice of techniques is determined. This can be expressed as

$$p_1^*x_1^a + p_2^*x_2^a = w_1x_1^a + w_2x_2^a,$$

or,

$$p^*x^a = wx^a, \qquad \text{(III-3)}$$

where p^* satisfies (II-5). Taking wx^a as a constant, one can illustrate (III-3), as before, in Figure 4. With reference to Figure 4, the following proposition may be interesting:

Proposition 3.

The surplus condition is equivalent to $w \in P^{DE}_{(wx^a)}$.

Proof.

One has only to show that w_1/w_2 is greater than $\min(p_1/p_2)$ and smaller than $\max(p_1/p_2)$. In fact

$$(1-bL_c)/a_2 > w_1/w_2 = 1/(1-a_1+a_2) > bL_c/(1-a_1)$$

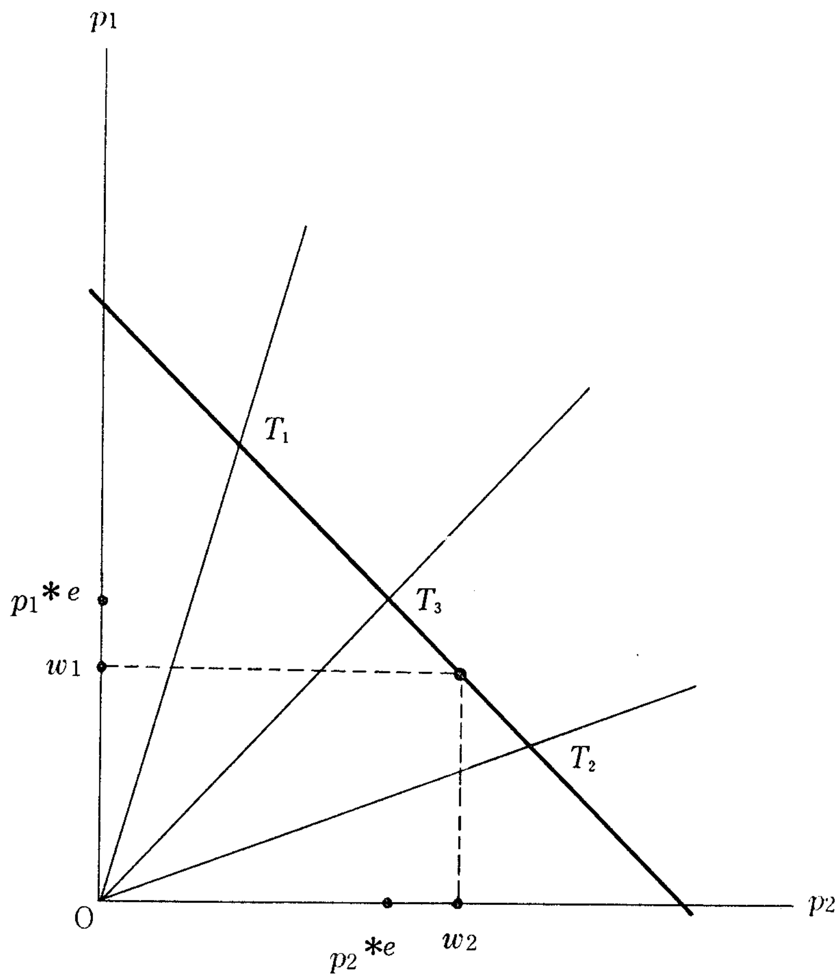
if and only if the surplus condition is true. Q. E. D.

In the case that $a_1 > a_2$, the point w lies on T_3T_2, because, as well-known, then $w_1 < p_1^{*e}$ and $w_2 > p_2^{*e}$, where (p_1^{*e}, p_2^{*e}) is an element of $P^E \cap P^{DE}_{(wx^a)}$.

It is well-known that the total profit is not equal to the total surplus value. The total profit is defined as

$$\text{total profit} = p^*x^a - p^*Mx^a,$$

Figure 4.



and the total surplus value as

$$\text{total surplus value} = wx^a - wMx^a.$$

Then, let us define the function:

$$F(p^*, \ x^a) = (p^*x^a - p^*Mx^a) - (wx^a - wMx^a).$$

In view of (III-3), this will be reduced to

$$H(p_1^*) = \{bLx^a(x_1{}^a/x_2{}^a) - A_1x^a\}\, p_1^* - bLx^a \cdot wx^a/x_2{}^a + wMx^a,$$

where $A_1 = (a_1, \ a_2)$.

Here, it is necessary to distinguish the three cases with respect to x^a. Let us introduce the following two sets:

$$S_{GE} = \bigcup_{\gamma \in \Gamma} \{x | x > 0, \text{ and } g > 0 \text{ in (III-1).}\}$$

$$S_{EGE} = \bigcup_{\gamma \in \Gamma} \{x | x > 0, \text{ and } g_1 = g_2 > 0 \text{ in (III-1).}\}$$

where $\Gamma = [0, \ 1 - bL_c(1 + a_2/1 - a_1))$.

The von Neumann path of the economy can be determined as the state with $\gamma=0$ and $g_1=g_2$ in (III–1). These sets being illustrated by polyhederal cones in R^2_+, the von Neumann path is one of the boundaries of S_{EGE}. Let $S_{EGE}(0)$ be the von Neumann path. Then, as discussed in Fujimori (1978), S_{GE} can be divided into three subsets, i.e. $S_{GE}-S_{EGE}$, $S_{EGE}(0)$ and $S_{EGE}-S_{EGE}(0)$.

Now, one can prove:

Proposition 4.

Suppose that $a_1>a_2$. Then,

i) $\partial H/\partial p_1>0 \iff x^a\in S_{GE}-S_{EGE}$,

ii) $\partial H/\partial p_1=0 \iff x^a\in S_{EGE}(0)$,

iii) $\partial H/\partial p_1<0 \iff x^a\in S_{EGE}-S_{EGE}(0)$.

Proof.

In fact, one gets

$$\partial H/\partial p_1=bLx^a(x_1{}^a/x_2{}^a)-A_1x^a,$$

Hence, $$\partial H/\partial p_1 \gtreqless 0 \text{ as } x_1{}^a/x_2{}^a \gtreqless A_1x^a/bLx^a.$$

The conclusion follows at once. Q. E. D.

That is to say, whether F is increasing or decreasing with respect to $p_1{}^*$ depends on x^a.

Based on the above arguments, one can make some comments on Marxian theory of disequilibrium growth and disequilibrium pricing.

Suppose that an initial point, x^0, is given in S_{EGE} and that the growth locus of the economy runs, crossing the von Neumann path, from the initial point to another in $S_{GE}-S_{EGE}$. While x^a remains in S_{EGE}, the price raise in dept. I decreases comparatively the profit-surplus value difference in view of Proposition 4. If x^a lies in $S_{GE}-S_{EGE}$, however, the price raise in capital goods will increase the profit-surplus value difference.

Therefore, it may be reasonable to consider that the equilibrium price system tends to hold while x^a is in S_{EGE}, even if the actual path is in the state of uneven growth moving towards $S_{GE}-S_{EGE}$. In $S_{GE}-S_{EGE}$, however, the superiority of dept. I in growth will be prompted by the price raise in capital goods. In this sense, the region $S_{GE}-S_{EGE}$ may be called the disequilibrium region.

IV. Concluding remarks

Thus far, the framework of a discussion that the disequilibrium growth accompanied by the superiority of dept. I to dept. II is one of the forms of motion in which the law of value asserts itself was made clear to a certain extent.

The picture of the economy in the state of uneven growth given above shows that the disequilibrium growth with $g_1 > g_2$ is one of the most vital phases in business cycles. And, the disequilibrium growth together with disequilibrium pricing will visit the economy when it arrives at $S_{GE} - S_{EGE}$.

The conclusions added in this paper are that equilibrium prices tend to be established in S_{EGE}, and that disequilibrium pricing may accompany the uneven growth in $S_{GE} - S_{EGE}$.

If based on the theory of value, this uneven growth of the economy which appears through competition will be found dependent on the conflict between the two major classes.

It must be noted again, however, that the whole discussion still rests on the presupposition that a_1 is greater than a_2. Then, it may be interesting to make some remarks on how arguments should be changed when one has $a_1 < a_2$: this case is not argued by most of Japanese Marxian economists.

One can define the sets, S_G, S_{GE}, S_{EGE}, P^{DE} and P^E in the same fashion as before, because the definitions of them are independent from the assumption. So, the propositions concerning them are also valid in this case. While, some values of Q_m, Q_M, γ_0 and so on should be altered, because they depend on the assumption.

Moreover, some propositions on the transformation of values into prices should be changed. Hence, the sign of the profit-surplus value difference will be changed. Namely, instead of Proposition 3(ii) in Fujimori (1978), one has: If $a_1 < a_2$, then $p_1^{*e} < w_1$ and $p_2^{*e} > w_2$. Therefore, the roles played by S_{EGE} and $S_{GE} - S_{EGE}$ should be interchanged in Proposition

4 and 5 in Fujimori (1978), if one has $a_1 < a_2$.

Accordingly, the most vital phase of uneven growth will be found in the locus starting from a point in $S_{GE} - S_{EGE}$ to another in S_{EGE}. The rest of the discussion may be repeated in the similar fashion.

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