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A Study on Robustly Stabilizable Controller for Pneumatic Servo System

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Abstract. This paper considers the finite- dimensional robustly stabilizing controller design for the pneumatic servo system whose transfer function takes the form of a strictly proper rational function times a delay. The designing controllers is based on H_∞-control theory.

Ⅰ. Introduction

In this paper we consider the robust stability problem in the framework of the H_∞-control . We take the pneumatic servo system as an infinite-dimensional mathematical model. We express modeling errors ,due to the change of values of time delay, as perturbations of the nominal model and find a fixed controller based on this nominal plant model, which stabilizes not only the nominal plant but also the family of perturbed plants in the closed loop system. We consider factor coprime perturbations ([1],[6],[7],[8],[13]) so that it is possible to find upper bounds of perturbations for the time delay and also for other some parameters to which the robustness is achieved. For the practical use it is desirable to have a finite-dimensional controller, so we take a finite-dimensional nominal plant and design the controller based on this nominal plant model. All the calculations for designing controllers , in which we use the state space techniques ([4]), are in finite-dimensional ones.

This paper is organized as follows. Section Ⅱ presents brief illustrations of the pneumatic servo system. In Section Ⅲ some background material in H_∞-control theory is reviewed which is needed for applying the H_∞-control to the control of the pneumatic servo system. In Section Ⅳ we apply the facts in Section Ⅲ to designing of finite-dimensional robustly stabilizable controllers for the pneumatic servo system.

Ⅱ. Pneumatic servo system

Figure1. shows the schematic diagram of the pneumatic servo system which is the control object in this paper. According prepared experiments and considering the applied voltage for power amplification as the input and the piston position as the output, respectively, the transfer function of the plant , in which the action delay of the piston and the computation-time of the computer are taken into account, can be described as follows ([10],[11],[15]):

$$P(s) = \frac{A\alpha e^{-Ls}}{s(D+Ms)(1+\tau s)}$$

where

A : pressure area of the piston $[cm^2]$,
M : load mass $[Kg]$,
D : viscosity friction parameter $[N \cdot s / cm^2]$,
α : transformation gain,
τ : time constant $[s]$,
L : time delay $[s]$.

The transfer function takes the form of a strictly proper rational function times a delay which has a pole on the imaginary axis.

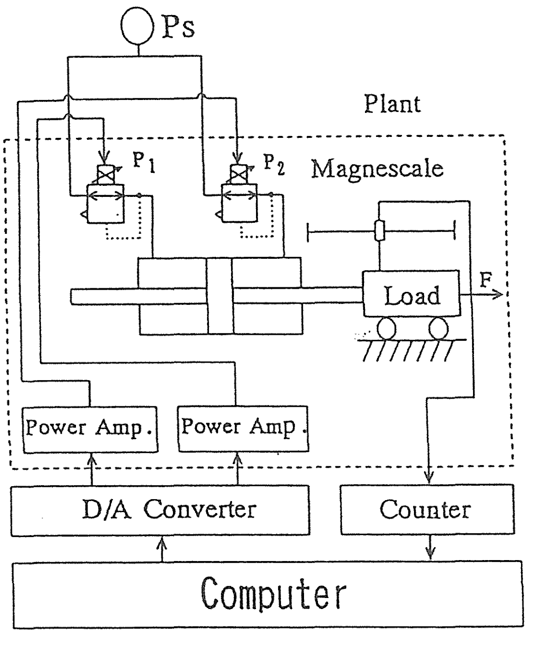

Figure1.
Schematic diagram of the pneumatic servo system

Ⅲ. Robust control under coprime factor perturbations

Notation. H_∞ denotes the Hardy space of bounded analytic functions in the right half plane of the complex plane, and $\|\cdot\|_\infty$ denotes its norm. F_∞ denotes the quotient field of H_∞. For the notational convenience, we will suppress matrix dimensions and use H_∞ and F_∞ also for the corresponding spaces of vector- and matrix-valued functions. If $P(s)$ is a matrix function of s, $P(s)^* := \overline{P(-\bar{s})}$.

If H and K are Hilbert spaces and $P: H \to K$ is an operator, then P^* denotes its adjoint and $\| P \|$ denotes its norm. Γ denotes the Hankel operator.

1 . Closed loop stability

Consider the feedback configuration of Figure2, where P denotes the plant to be controlled and C represents the controller to be designed.

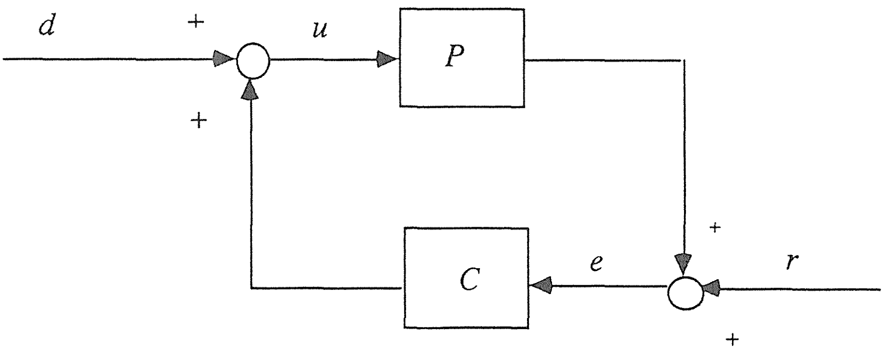

Figure 2. Standard feedback configuration

Let the feedback configuration of Figure2, where $P \in F_\infty$ and $C \in F_\infty$ be denoted by $[P, C]$. The configuration is defined to be stable if the operators $u \rightarrow d$ and $e \rightarrow r$ are bounded. This is equivalent to

$$\begin{pmatrix} I & -C \\ -P & I \end{pmatrix}^{-1} = \begin{pmatrix} (I-CP)^{-1} & C(I-PC)^{-1} \\ P(I-CP)^{-1} & (I-PC)^{-1} \end{pmatrix}$$

belonging to H_∞ . If $[P, C]$ is stable , then we say that the controller C stabilizes the plant P.

2. Controller parametrization

We will assume that the transfer function $P(s)$ of the plant P admits right and left coprime factorizations as follows:

$$P(s) = N(s)M^{-1}(s) = \tilde{M}^{-1}(s)\tilde{N}(s)$$

where $M, N, \tilde{M}, \tilde{N} \in H_\infty$ such that there exist $X, Y, \tilde{X}, \tilde{Y} \in H_\infty$ satisfying the Bezout identity

$$\tilde{M}X + \tilde{N}Y = \tilde{X}M + \tilde{Y}N = I$$

In fact existence of such a factorization is necessary for the existence of a stabilizing controller belonging to F_∞ [10].

NM^{-1}, $\tilde{M}^{-1}\tilde{N}$ is called a normalized right and left coprime factorization of P respectively if it is a coprime factorization and

$$M^*(-j\omega)M(j\omega) + N^*(-j\omega)N(j\omega) = I$$
$$\tilde{M}(j\omega)\tilde{M}^*(-j\omega) + \tilde{N}(j\omega)\tilde{N}^*(-j\omega) = I \qquad \text{for all real } \omega.$$

It is known that if P has a right and left coprime factorization, then it has a normalized one[14].

Let P be a plant with normalized coprime factorization $P = NM^{-1} = \tilde{M}^{-1}\tilde{N}$. Then there exist $U, V, \tilde{U}, \tilde{V} \in H_\infty$ such that

$$\begin{pmatrix} \tilde{V} & -\tilde{U} \\ -\tilde{N} & \tilde{M} \end{pmatrix}\begin{pmatrix} M & U \\ N & V \end{pmatrix} = \begin{pmatrix} M & U \\ N & V \end{pmatrix}\begin{pmatrix} \tilde{V} & -\tilde{U} \\ -\tilde{N} & \tilde{M} \end{pmatrix} = \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix}$$

It is well known that all controllers which stabilize P can be parametrized in the form

$$C = (U + MQ)(V + NQ)^{-1} = (\tilde{V} + Q\tilde{N})^{-1}(\tilde{U} + Q\tilde{M}) \qquad \text{for some} \quad Q \in H_\infty.$$

4. Robust controller and robustness margin under coprime factor perturbations

The robust design objective is to find a feedback controller C which stabilizes not only the nominal plant P, but also the family of perturbed plants defined by

$$\{P_1\} = \{(\tilde{M} + \Delta_{\tilde{M}})^{-1}(\tilde{N} + \Delta_{\tilde{N}}) \text{ where } \Delta_{\tilde{M}}, \Delta_{\tilde{N}} \in H_\infty \text{ and } \left\|\left(\Delta_{\tilde{M}}, \Delta_{\tilde{N}}\right)\right\|_\infty < b\} \text{ (see Figure3)}$$

If there exists a controller C for P which stabilizes the family $\{P_1\}$ of perturbed plants, then $\{P_1\}$ is said to be factor robustly stabilizable with factor robustness margin b.

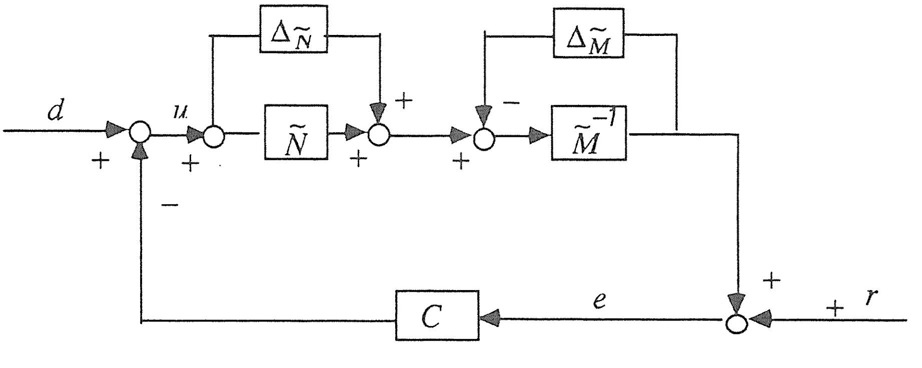

Figure 3. Left coprime factor perturbations

The following result was obtained by Vidyasagar and Kimura for finite dimensional systems in [13], and which is extended to for infinite dimensional systems by Georgiou and Smith in [6].

Theorem1: Let C be a controller with transfer function $C=(U+MQ)(V+NQ)^{-1}$ for the plant $P=\tilde{M}^{-1}\tilde{N}$. Then the following are equivalent.

a) $[P_1,C]$ is stable for all P_1 with transfer function $P_1=(\tilde{M}+\Delta_{\tilde{M}})^{-1}(\tilde{N}+\Delta_{\tilde{N}})$ where $\Delta_{\tilde{M}}$, $\Delta_{\tilde{N}}$ $\in H_\infty$ and $\left\| \left(\Delta_{\tilde{M}},\Delta_{\tilde{N}}\right) \right\|_\infty < b$.

b) $\left\| \begin{pmatrix} U \\ V \end{pmatrix} + \begin{pmatrix} M \\ N \end{pmatrix} Q \right\|_\infty \leq \frac{1}{b}$

A dual result holds for normalized right coprime factor perturbations.

Let b_{opt} denote the supremum over all b such that there exists a controller C which stabilizes all plants P_1 with transfer function $P_1=(\tilde{M}+\Delta_{\tilde{M}})^{-1}(\tilde{N}+\Delta_{\tilde{N}})$ where $\Delta_{\tilde{M}}$ $\Delta_{\tilde{N}}$ $\in H_\infty$ and $\left\| \left(\Delta_{\tilde{M}},\Delta_{\tilde{N}}\right) \right\|_\infty < b$. It follows that

$$b_{opt}^{-1} = \inf_{Q\in H_\infty} \left\| \begin{pmatrix} U \\ V \end{pmatrix} + \begin{pmatrix} M \\ N \end{pmatrix} Q \right\|_\infty .$$

It is also known that this infimum is achieved for some $Q_0 \in H_\infty$. We say that the controller $C_0=(U+MQ_0)(V+NQ_0)^{-1}$ is the maximum robust controller for P.

The following formula for the maximum factor robustness margin b_{opt} was given in [6].

Theorem 2: The maximum factor robustness margin b_{opt} for $P=\tilde{M}^{-1}\tilde{N}=NM^{-1}$ is given by

$$b_{opt} = \left(1 - \left\| \Gamma\begin{pmatrix} \tilde{M}^* \\ -\tilde{N}^* \end{pmatrix} \right\|^2 \right)^{\frac{1}{2}} .$$

The following approximation result is useful for designing finite-dimensional robustly stabilizing controllers. [1]

Theorem3: Suppose that $P=P_r+P_h$ where P_r is rational and contains all the unstable poles and $P_h \in H_\infty$ with $\| P_h \|_\infty < \mu \,(<\varepsilon)$.
Then if C stabilizes P_r with factor robustness margin ε, it stabilizes P with factor robustness margin at least equal to $\varepsilon-\mu$.

Ⅳ. Design of finite-dimensional robustly stabilizing controller for the pneumatic servo system

In the pneumatic servo system the transfer function is given by

$$P(s) = \frac{Ke^{-Ls}}{s(D+Ms)(1+\tau s)} \quad \text{where} \quad K = A\alpha \ ,$$

which belongs F_∞.

We consider an uncertain system with changing values of parameters described by

$$\tilde{P}(s) = \frac{Ke^{-\tilde{L}s}}{s(\tilde{D}+\tilde{M}s)(1+\tau s)} \quad \text{where } 0 \le \tilde{L} \le L \quad D_1 \le \tilde{D} \le D_2 \quad M_1 \le \tilde{M} \le M_2 .$$

For designing a finite-dimensional robustly stabilizing controller, first, ignoring the time delay we take the simple nominal plant as follows:

$$P_0(s) = \frac{K}{s(D_0+M_0 s)(1+\tau s)} \quad \text{where} \quad D_0 = \frac{D_1+D_2}{2} \text{ and } M_0 = \frac{M_1+M_2}{2} .$$

Obtaining a minimum realization for P_0 and solving the algebraic Riccati equations ,we have the maximum robustness margin b_{opt} and the normalized coprime factorization of P_0 such that

$$P_0(s) = \frac{K/p(s)}{s(D_0+M_0 s)(1+\tau s)/p(s)} = \frac{n(s)}{m(s)} \quad \text{where } p(s) \text{ is a polynomial of degree3.}$$

Then the perturbed plant is described by

$$P_\Delta(s) = \frac{n(s)+\Delta_n(s)}{m(s)+\Delta_m(s)}$$

with the factor perturbations $\Delta_n(s) = n(s)\left(1-e^{-\Delta_L s}\right)$ and $\Delta_m(s) = \dfrac{s(\Delta_D+\Delta_M s)(\tau s+1)}{p(s)}$.

We also find the maximum robust controller C_0 for P_0.

If $\delta = \left\|\Delta_n(s)\right\|_\infty < b_{opt}$ for $\Delta_L = L$, the maximum robust controller C_0 serves a robustly stabilizing controller for the perturbed plants such that

$$\tilde{P}(s) = \frac{Ke^{-\tilde{L}s}}{s(\tilde{D}+\tilde{M}s)(1+\tau s)}$$

with $0 \le \tilde{L} \le L$, $M_0 - \delta_1 \le \tilde{M} \le M_0 + \delta_1$ and $D_0 - \delta_2 \le \tilde{D} \le D_0 + \delta_2$

where δ_1 and δ_2 are calculated by $\left\|\Delta_m(s)\right\|_\infty \le \sqrt{b_{opt} - \delta^2}$.

If $\delta = \left\|\Delta_n(s)\right\|_\infty \ge b_{opt}$ for $\Delta_L = L$, in this case, using the nominal values M_0, D_0, and

$L_0 = \frac{L}{2}$ we take the nominal plant as follows:

$$P_0(s) = \frac{Ke^{-L_0 s}}{s(D_0 + M_0 s)(1+\tau s)}.$$

Now we have the normalized coprime factorization of P_0 such that

$$P_0(s) = \frac{Ke^{-L_0 s} / p(s)}{s(D_0 + M_0 s)(1+\tau s) / p(s)} = \frac{e^{-L_0 s} n(s)}{m(s)}$$ where $p(s)$ is a polynomial of degree3.

Then the perturbed plant is described by

$$P_\Delta(s) = \frac{e^{-L_0 s} n(s) + \Delta_n(s)}{m(s) + \Delta_m(s)}$$

with the factor perturbations $\Delta_n(s) = e^{-L_0 s} n(s)\left(1 - e^{-\Delta_L s}\right)$ and $\Delta_m(s) = \frac{s(\Delta_D + \Delta_M s)(\tau s + 1)}{p(s)}$.

Taking an appropriate order modal approximation, we have $P = P_r + P_h$ with $\|P_h\|_\infty < \mu$. And calculate the maximum robustness margin ε and the maximum robust controller C_r for P_r. If $\varepsilon - \mu$ is acceptable , by Theorem3, the maximum robust controller C_r for P_r also serves as a robust controller for the original infinite-dimensional system P with factor robustness margin of at least $\varepsilon - \mu$.

Hence, the maximum robust controller C_r is a stabilizing controller for the perturbed plants such that

$$\tilde{P}(s) = \frac{Ke^{-\tilde{L}s}}{s(\tilde{D} + \tilde{M}s)(1+\tau s)}$$

with $0 \le \tilde{L} \le L$, $M_0 - \delta_1' \le \tilde{M} \le M_0 + \delta_1'$ and $D_0 - \delta_2' \le \tilde{D} \le D_0 + \delta_2'$.

Note that in the above procedure of designing the controller all the calculations are finite-dimensional ones.

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