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Construction of a system of differential operators as annihilators of a cohomology class –in connection with quasihomogeneous singularities–

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We studied the residue pairing induced by an algebraic local cohomology class from a view point of the theory of D-modules in [2] and [3]. For a cohomology class of one dimensional case, we constructed a linear differential operator of order one which was the theoretical foundation of an algorithm for computing residues (cf. [3]). On the other hand, in the theory of quasihomogeneous singularities, it is known that linear partial differential operators of order one determined by weights play an important role.

In this paper, we look at a differential operaots of order 1 associated to an algebraic local cohomology class. First, we consider the normal forms of quasihomogeneous polynomials. Then we provide a method for computing a presentation of a cohomology class for a quasihomogeneous polynomial. Next, we provide a method for computing linear partial differential operators of order one associated to a cohomology class of general n dimensional case.

1 Preliminaries

In this section, we recall some basic results about a characterization of a cohomology class obtained in [2]. Let $X = \mathbf{C}^n$ and $\mathcal{O}_X$ the sheaf of holomorphic functions on X. Let $f_1, \ldots, f_n$ be a regular sequence of holomorphic functions of $z = (z_1, \ldots, z_n)$ in X. Denote by I the sheaf of ideal generated by $f_1, \ldots, f_n$ over $\mathcal{O}_X$.

Let i be the canonical mapping $i : \mathcal{E}xt^n_{\mathcal{O}_X}(\mathcal{O}_X/I, \mathcal{O}_X) \to \mathcal{H}^n_{[A]}(\mathcal{O}_X)$, where $A = \{z \in X | f_1 = \cdots = f_n = 0\}$. Let us denote by $\left[\begin{array}{c} 1 \\ f_1 \cdots f_n \end{array}\right]$ the element of $\mathcal{E}xt^n_{\mathcal{O}_X}(\mathcal{O}_X/I, \mathcal{O}_X)$ associated to the meromorphic function $1/f_1 \cdots f_n$.

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Put $m = i\left(\begin{bmatrix} 1 \\ f_1 \cdots f_n \end{bmatrix}\right) \in \mathcal{H}^n_{[A]}(\mathcal{O}_X)$. Assume that A consists of finitely many points $A_j \quad (j = 1, \ldots, \nu)$. Then, there exist $m_j \in \mathcal{H}^n_{[A_j]}(\mathcal{O}_X)$, $j = 1, \ldots, \nu$ such that $m = m_1 + \cdots + m_\nu$. Since $\mathcal{H}^n_{[A_j]}(\mathcal{O}_X)$ is a simple holonomic $\mathcal{D}_X$-module, we have $\mathcal{D}_X m_j = \mathcal{H}^n_{[A_j]}(\mathcal{O}_X)$. Denote by $\mathcal{A}nn$ the annihilator ideal of m as $\mathcal{D}_X$-module. The construction of this ideal $\mathcal{A}nn$ is important as to study the Grothendieck local residue with the viewpoint of $\mathcal{D}_X$-module. We have the following theorem.

Theorem 1
([2], [3], [4]) *At each point A_j, we have*

$$\{\phi(z) \in \mathcal{H}^n_{[A_j]}(\mathcal{O}_X) \mid R\phi(z) = 0, {}^\forall R \in \mathcal{A}nn\} = \{cm_j \mid c \in \mathbf{C}\}, j = 1, \ldots, \nu.$$

This asserts that the cohomology class m_j can be characterized as a solution of a system of linear differential equations up to the constant factor.

2 The cohomology class associated to quasihomogeneous singularities

In the theory of quasihomogeneous singularities, it is known that there exist annihilators of order one determined by weights. Let $f(z) = f(z_1, \ldots, z_n)$ be a quasihomogeneous polynomial of degree d with weights α_1, ..., α_n, that is, for any $\lambda > 0$ we have

$$f(\lambda^{\alpha_1} z_1, \ldots, \lambda^{\alpha_n} z_n) = \lambda^d f(z).$$

Let us consider a cohomology class m associated to the meromorphic function $1/\prod_{j=1}^n f_j$ where $f_j := \partial f/\partial z_j$, $j = 1$, ..., n. Denote by d_j the quasidegree of f_j. Now we have the following fact:

FACT *The annihilator ideal $\mathcal{A}nn$ of m is generated by F_1, ..., F_n and P_1, ..., P_ℓ where $F_j = f_j$ and P_j is given by a linear combination of $\alpha_j \partial_j + d_j$, $j = 1$, ..., $\ell \le n$.*

By virtue of Theorem 1, we describe a method for computing a presentation of the cohomology class.

For the ideal $I = \langle f_1, \ldots, f_n \rangle$, there exist one variable functions of each coordinates z_1, ..., z_n belong to I. These functions are of the forms $z_j^{r_j}$, $j = 1$, ..., n. Assume that $r_j := \{r : z_j^r \in I\} > 1$ for some j.

- Put $m = \sum a_\gamma z^\gamma / z^r$ with $a_\gamma \in \mathbf{Q}$, $z^\gamma := z_1^{\gamma_1} \cdots z_n^{\gamma_n}$, $z^r := z_1^{r_1} \cdots z_n^{r_n}$ for $\gamma = (\gamma_1, \ldots, \gamma_n)$, $\gamma_j < r_j$ $(j = 1, \ldots, n)$.
- Equations $P_j m = 0$, $1 \le j \le \ell$, $F_k m = 0$, $1 \le k \le n$, determine m up to a constant fuctor.

- $Jac := (\partial(f_1, \ldots, f_n)/\partial(z_1, \ldots, z_n))$, *mult* be the multiplicity of the origin as commono zero of f_1, …, f_n and δ the dirac's delta function at origin.
- The formula $Jac \cdot m = mult \cdot \delta$ determine m uniquely.

We draw up lists of generators of annihilator ideal $\mathcal{A}nn$ and presentations of cohomology classes associated to normal forms of quasihomogeneous polynomials in Appendix.

Let us consider a sequence f_1, …, f_n which provides semiquasihomogeneous polynomials with the same weights. In the next example, we can see there are some relation between the weights and a operator which is one of generators of $\mathcal{A}nn$.

Example 1
Let $X = \{(x, y)|x, y \in \mathbf{C}\}$, $f_1(x, y) = x^5$, $f_2(x, y) = -8yx^4 + 2x^3 - 4yx^2 + y^2$. f_1 *and* f_2 *provide semiquasihomogeneous polynomials with weight* $(\underline{2}, \underline{3})$ *and quasidegrees* $\underline{10}$ *and* $\underline{6}$. *The ideal* $\mathcal{A}nn$ *of the cohomology class* $m = i\left(\left[\begin{array}{c}1\\ f_1f_2\end{array}\right]\right) \in \mathcal{H}^2_{[(0,0)]}(\mathcal{O}_X)$ *is generated by* $F_1 = x^5$, $F_2 = -8yx^4 + 2x^3 - 4yx^2 + y^2$ *and* $P = (-15x^3 + 4x^2 + \underline{6x)\partial_x} + (16x^3 + 6x^2 + \underline{9y)\partial_y} - 75x^2 + 20x + \underline{48}$ *with* $\partial_x := \partial/\partial x$ *and* $\partial_y := \partial/\partial y$. *The under lined parts are written in the form* $3(2x\partial_x + 10) + 3(3y\partial_y + 6)$.

3 Annihilators of order one

In view of the isomorphism

$$\mathcal{H}^n_{[Y]}(\mathcal{O}_X) \cong \mathcal{D}_X/\mathcal{A}nn,$$

an algorithm for computing $\mathcal{H}^n_{[Y]}(\mathcal{O}_X)$ is provided in [1] and this algorithm was implemented by using *Kan* ([5]). When given regular sequence of holomorphic functions is not so complicated, this algorithm is carried out and provide proper operators which generate $\mathcal{A}nn$. But given sequence become complicated, this procedure is often too slow or fails.

On the other hand, it would suffice to find annihilators of order zero and order one for characterizing $\mathcal{A}nn$. Now we can obtain annihilators of order 1 by the computation of syzygies on the basis of the isomorphism

$$\mathcal{H}^n_{[Y]}(\mathcal{O}_X) \cong \frac{\mathcal{O}_X[*Y_1 \cup \cdots \cup Y_n]}{\sum_{i=1}^n \mathcal{O}_X[*Y_1 \cup \cdots \cup Y_{i-1} \cup \widehat{Y_i} \cup Y_{i+1} \cup \cdots \cup Y_n]}, \tag{1}$$

where $Y_j = \{z \in X | f_j(z) = 0\}$ and $\mathcal{O}_X[*A]$ stands for a meromorphic function with poles at A. For an differential operator $R = a_0 + a_1\partial_1 + \cdots + a_n\partial_n$ of order 1 and the function $M = 1/f_1 \cdots f_n$, we have

$$RM = \frac{-a_1f_{11} - \cdots - a_nf_{1n}}{f_1^2 f_2 \cdots f_n} + \cdots + \frac{-a_1f_{n1} - \cdots - a_nf_{nn}}{f_1 \cdots f_{n-1}f_n^2} + \frac{a_0}{f_1 \cdots f_n},$$

where $f_{ij} = \partial f_i/\partial z_j$. Then, for functions $a_1, \ldots, a_n, c_{ij}$ such that

$$\begin{cases} -a_1 f_{11} - \cdots - a_n f_{1n} = c_{11} f_1 + \cdots + c_{1n} f_n + \{\text{higher order of } f_1, \ldots, f_n\}, \\ \cdots \\ -a_1 f_{n1} - \cdots - a_n f_{nn} = c_{n1} f_1 + \cdots + c_{nn} f_n + \{\text{higher order of } f_1, \ldots, f_n\}, \end{cases}$$

R provide an annihilator of the cohomology class m associated to M with $a_0 = -c_{11} - \cdots - c_{nn}$. These functions are given by computing syzygies. That is, $(a_1, \ldots, a_n, c_{11}, c_{12}, \ldots, c_{nn})$ are the sygyzies of the $n(n+1)$-tuple

$$\begin{pmatrix} f_{11} \\ f_{21} \\ \cdots \\ f_{n1} \end{pmatrix}, \ldots, \begin{pmatrix} f_{1n} \\ f_{2n} \\ \cdots \\ f_{nn} \end{pmatrix}, \begin{pmatrix} f_1 \\ 0 \\ \cdots \\ 0 \end{pmatrix}, \ldots, \begin{pmatrix} f_n \\ 0 \\ \cdots \\ 0 \end{pmatrix}, \ldots, \begin{pmatrix} 0 \\ \cdots \\ 0 \\ f_1 \end{pmatrix}, \ldots, \begin{pmatrix} 0 \\ \cdots \\ 0 \\ f_n \end{pmatrix}.$$

4 Appendix

For a normal form f of a quasihomogeneous polynomial, we obtain generators of the annihilator ideal Ann of the cohomology class associated to $1/\prod_j f_j$ where $f_j := \partial f/\partial z_j$. Hence, we can compute the presentation of the cohomology class by a method in section 2. We draw up lists of generators of the annihilator ideal Ann and the presentation of the cohomology classes for normal forms of quasihoomogeneous polynomials.

type of singularity, weight	quasidegree	normal form
f_x	quasidegree	f_1
f_y	quasidegree	f_2
f_z	quasidegree	f_3
Ann		generators
cohomology	quasidegree	

(b, a)	ab	$f = x^a + y^b$
f_x	$b(a-1)$	$f_1 = ax^{a-1}$
f_y	$a(b-1)$	$f_2 = by^{b-1}$
Ann		$x^{a-1}, y^{b-1}, x\partial_x + a - 1, y\partial_y + b - 1$
cohomology	$-(2ab - a - b)$	$\frac{1}{ab}\frac{1}{x^{a-1}y^{b-1}}$

E_7 $(3,2)$	9	$f = x^3 + y^3 x$
f_x	6	$f_1 = 3x^2 + y^3$
f_y	7	$f_2 = 3y^2 x$
Ann		$x^3, y^2 x, 3x^2 + y^3, 3x\partial_x + 2y\partial_y + 13$
cohomology	-13	$\frac{-1}{3}\frac{1}{xy^5} + \frac{1}{9}\frac{1}{x^3y^2}$

D_k $(k-2, 2)$	$2k-2$	$f = x^2 y + y^{k-1}$
f_x	k	$f_1 = 2xy$
f_y	$2k-4$	$f_2 = x^2 + (k-1)y^{k-2}$
Ann		$x^3, yx, x^2 + (k-1)y^{k-2}, (k-2)x\partial_x + 2y\partial_y + 3k - 4$
cohomology	$-(3k-4)$	$\frac{1}{2(k-1)}\frac{1}{xy^{k-1}} + \frac{-1}{2}\frac{1}{x^3y}$

(α,β,γ) $a\alpha=b\beta=c\gamma$	6	$f=x^a+y^b+z^c$
f_x	$(a-1)\alpha$	$f_1=ax^{a-1}$
f_y	$(b-1)\beta$	$f_2=by^{b-1}$
f_z	$(c-1)\gamma$	$f_3=cz^{c-1}$
Ann		$x^{a-1},y^{b-1},z^{c-1},$ $x\partial_x+a-1,y\partial_y+b-1,z\partial_z+c-1$
cohomology	$-((a-1)\alpha+(b-1)\beta+(c-1)\gamma)$	$\frac{1}{(a-1)(b-1)(c-1)}\frac{1}{x^{a-1}y^{b-1}z^{c-1}}$

E_{13} $(15,10,4)$	30	$f=x^2+y^3+yz^5$
f_x	15	$f_1=2x$
f_y	20	$f_2=3y^2+z^5$
f_z	26	$f_3=5z^4y$
Ann		$x,y^3,z^4y,3y^2+z^5,5y\partial_y+2z\partial_z+23$
cohomology	-61	$\frac{-1}{10}\frac{1}{xyz^9}+\frac{1}{30}\frac{1}{xy^3z^4}$

Z_{11} $(15,6,8)$	30	$f=x^2+y^5+yz^3$
f_x	15	$f_1=2x$
f_y	24	$f_2=5y^4+z^3$
f_z	22	$f_3=3z^2y$
Ann		$x,y^5,z^2y,5y^4+z^3,3y\partial_y+4z\partial_z+23$
cohomology	-61	$\frac{-1}{6}\frac{1}{xyz^5}+\frac{1}{30}\frac{1}{zy^5z^2}$

Z_{12} $(11,6,4)$	22	$f=x^2+y^3z+yz^4$
f_x	11	$f_1=2x$
f_y	16	$f_2=3zy^2+z^4$
f_z	18	$f_3=y^3+4z^3y$
Ann		$x,y^5,zy^3,y^3+4z^3y,3zy^2+z^4,3y\partial_y+2z\partial_z+17$
cohomology	-45	$\frac{-3}{22}\frac{1}{xyz^7}+\frac{1}{22}\frac{1}{xy^3z^4}+\frac{-2}{11}\frac{1}{xy^5z}$

Z_{13} $(9,3,5)$	18	$f=x^2+y^6+yz^3$
f_x	9	$f_1=2x$
f_y	15	$f_2=6y^5+z^3$
f_z	13	$f_3=3z^2y$
Ann		$x,y^6,z^2y,6y^5+z^3,3y\partial_y+5z\partial_z+28$
cohomology	-37	$\frac{-1}{6}\frac{1}{xyz^5}+\frac{1}{36}\frac{1}{xy^6z^2}$

W_{13} $(8,4,3)$	16	$f=x^2+y^4+yz^4$
f_x	8	$f_1=2x$
f_y	12	$f_2=4y^3+z^4$
f_z	13	$f_3=4z^3y$
Ann		$x,y^4,z^3y,4y^3+z^4,4y\partial_y+3z\partial_z+25$
cohomology	-33	$\frac{-1}{8}\frac{1}{xyz^7}+\frac{1}{32}\frac{1}{xy^4z^7}$

Q_{10} $(8,6,9)$	24	$x^3+y^4+yz^2$
f_x	16	$f_1=3x^2$
f_y	18	$f_2=4y^3+z^2$
f_z	15	$f_3=3zy$
Ann		$x^2,y^4,zy,4y^3+z^2,x\partial_x+2,2y\partial_y+3z\partial_z+11$
cohomology	-49	$\frac{-1}{9}\frac{1}{x^2yz^3}+\frac{1}{36}\frac{1}{x^2y^4z}$

Q_{11} (6, 4, 7)	18	$x^3+xy^3+yz^2$
f_x	12	$f_1=3x^2+y^3$
f_y	14	$f_2=3y^2x+z^2$
f_z	11	$f_3=2zy$
$\mathcal{A}nn$		$x^3, 3x^2+y^3, zx^2, zy, 3y^2x+z^2, 6x\partial_x+4y\partial_y+7z\partial_z+37$
cohomology	−37	$\frac{-1}{6}\frac{1}{xy^6z}+\frac{-1}{6}\frac{1}{x^2yz^3}+\frac{1}{18}\frac{1}{x^3y^3z}$

Q_{12} (5, 3, 6)	15	$x^3+y^5+yz^2$
f_x	10	$f_1=3x^2$
f_y	12	$f_2=5y^4+z^2$
f_z	9	$f_3=2zy$
$\mathcal{A}nn$		$x^2, y^5, zy, 5y^4+z^2, y\partial_y+2z\partial_z+7, x\partial_x+2$
cohomology	−31	$\frac{-1}{6}\frac{1}{x^2yz^3}+\frac{1}{30}\frac{1}{x^2y^5z}$

S_{11} (4, 6, 5)	16	$x^4+xy^2+yz^2$
f_x	12	$f_1=4x^3+y^2$
f_y	10	$f_2=2yx+z^2$
f_z	11	$f_3=zy$
$\mathcal{A}nn$		$x^4, 4x^3+y^2, zx^3, zy, 2yx+z^2, 4x\partial_x+6y\partial_y+5z\partial_z+33$
cohomology	−33	$\frac{-1}{2}\frac{1}{xy^4z}+\frac{-1}{4}\frac{1}{x^3yz^3}+\frac{1}{8}\frac{1}{x^4y^2z}$

S_{12} (1, 1, 1)	3	$x^2y+y^2z+z^2x$
f_x	2	$f_1=2yx+z^2$
f_y	2	$f_2=x^2+2zy$
f_z	2	$f_3=2zx+y^2$
$\mathcal{A}nn$		$x^4, yx^2, y^2x, x^3-y^3, 2zx+y^2, x^2+2zy, 2yx+z^2, x\partial_x+y\partial_y+z\partial_z+6$
cohomology	−6	$\frac{-2}{9}\frac{1}{xyz^4}+\frac{-2}{9}\frac{1}{xy^4z}+\frac{1}{9}\frac{1}{x^2y^2z^2}+\frac{-2}{9}\frac{1}{x^4yz}$

E_{19} (21, 14, 4)	42	$f=x^2+y^3+yz^7$
f_x	21	$f_1=2x$
f_y	28	$f_2=3y^2+z^7$
f_z	38	$f_3=7z^6y$
$\mathcal{A}nn$		$x, y^3, z^6y, 3y^2+z^7, 7y\partial_y+2z\partial_z+33$
cohomology	−87	$\frac{-1}{14}\frac{1}{xyz^{13}}+\frac{1}{42}\frac{1}{xy^3z^6}$

W_{17} (10, 5, 3)	20	$f=x^2+y^4+yz^5$
f_x	10	$f_1=2x$
f_y	15	$f_2=4y^3+z^5$
f_z	17	$f_3=5z^4y$
$\mathcal{A}nn$		$x, y^4, z^4y, 4y^3+z^5, 5y\partial_y+3z\partial_z+32$
cohomology	−42	$\frac{-1}{10}\frac{1}{xyz^9}+\frac{1}{40}\frac{1}{xy^4z^4}$

Z_{15} (7, 2, 4)	14	$f=x^2+y^7+yz^3$
f_x	7	$f_1=2x$
f_y	12	$f_2=7y^6+z^3$
f_z	10	$f_3=3z^2y$
$\mathcal{A}nn$		$x, y^7, z^2y, 7y^6+z^3, y\partial_y+2z\partial_z+11$
cohomology	−29	$\frac{-1}{6}\frac{1}{xyz^5}+\frac{1}{42}\frac{1}{xy^7z^2}$

Z_{17} (12,3,7)	24	$f = x^2 + y^8 + yz^3$
f_x	12	$f_1 = 2x$
f_y	21	$f_2 = 8y^7 + z^3$
f_z	17	$f_3 = 3z^2y$
Ann		$x, y^8, z^2y, 8y^7 + z^3, 3y\partial_y + 7z\partial_z + 38$
cohomology	-50	$\frac{-1}{6}\frac{1}{xyz^5} + \frac{1}{48}\frac{1}{xy^8z^2}$

Z_{18} (17,10,4)	34	$f = x^2 + y^3z + yz^6$
f_x	17	$f_1 = 2x$
f_y	24	$f_2 = 3zy^2 + z^6$
f_z	30	$f_3 = y^3 + 6z^5y$
Ann		$x, y^5, zy^3, y^3 + 6z^5y, 3zy^2 + z^6, 5y\partial_y + 2z\partial_z + 27$
cohomology	-71	$\frac{-3}{34}\frac{1}{xyz^{11}} + \frac{1}{34}\frac{1}{xy^3z^6} + \frac{-3}{17}\frac{1}{xy^5z}$

Z_{19} (27,6,16)	54	$f = x^2 + y^9 + yz^3$
f_x	27	$f_1 = 2x$
f_y	48	$f_2 = 9y^8 + z^3$
f_z	338	$f_3 = 3z^2y$
Ann		$x, y^9, z^2y, 9y^8 + z^3, 3y\partial_y + 8z\partial_z + 43$
cohomology	-113	$\frac{-1}{6}\frac{1}{xyz^5} + \frac{1}{54}\frac{1}{xy^9z^2}$

Q_{16} (7,3,9)	21	$f = x^3 + y^7 + yz^2$
f_x	14	$f_1 = 3x^2$
f_y	18	$f_2 = 7y^6 + z^2$
f_z	12	$f_3 = 2zy$
Ann		$x^2, y^7, zy, 7y^6 + z^2, y\partial_y + 3z\partial_z + 10, x\partial_x + 2$
cohomology	-44	$\frac{-1}{6}\frac{1}{x^2yz^3} + \frac{1}{42}\frac{1}{x^2y^7z}$

Q_{14} (4,2,5)	12	$f = x^3 + y^6 + yz^2$
f_x	8	$f_1 = 3x^2$
f_y	10	$f_2 = 6y^5 + z^2$
f_z	7	$f_3 = 2zy$
Ann		$x^2, y^6, zy, 6y^5 + z^2, 2y\partial_y + 5z\partial_z + 17, x\partial_x + 2$
cohomology	-25	$\frac{-1}{6}\frac{1}{x^2yz^3} + \frac{1}{36}\frac{1}{x^2y^6z}$

Q_{17} (10,4,13)	30	$f = x^3 + xy^5 + yz^2$
f_x	20	$f_1 = 3x^2 + y^5$
f_y	26	$f_2 = 5y^4x + z^2$
f_z	17	$f_3 = 2zy$
Ann		$x^2, y^5, zy, 5y^4x + z^2, 10x\partial_x + 4y\partial_y + 13z\partial_z + 63$
cohomology	-63	$\frac{-1}{10}\frac{1}{xy^{10}z} + \frac{-1}{6}\frac{1}{x^2yz^3} + \frac{1}{30}\frac{1}{x^3y^5z}$

Q_{18} (16,6,21)	48	$f = x^3 + y^8 + yz^2$
f_x	32	$f_1 = 3x^2$
f_y	42	$f_2 = 8y^7 + z^2$
f_z	27	$f_3 = 2zy$
Ann		$x^2, y^8, zy, 8y^7 + z^2, 2y\partial_y + 7z\partial_z + 23, x\partial_x + 2$
cohomology	-101	$\frac{1}{6}\frac{-1}{x^2yz^3} + \frac{-1}{48}\frac{-1}{x^2y^8z}$

S_{14} (2,4,3)	10	$f = x^5 + xy^2 + yz^2$
f_x	8	$f_1 = 5x^4 + y^2$
f_y	6	$f_2 = 2yx + z^2$
f_z	7	$f_3 = 2zy$
Ann		$x^5, 5x^4 + y^2, zx^4, zy, 2yx + z^2, 2x\partial_x + 4y\partial_y + 3z\partial_z + 21$
cohomology	-21	$\frac{-1}{4}\frac{1}{xy^4z} + \frac{-1}{10}\frac{1}{x^4yz^3} + \frac{1}{20}\frac{1}{x^5y^2z}$

S_{16} (5,7,3)	17	$f = x^2y + y^2z + z^4x$
f_x	12	$f_1 = 2yx + z^4$
f_y	10	$f_2 = x^2 + 2zy$
f_z	14	$f_3 = 4z^3x + y^2$
Ann		$x^4, yx^2, y^2x, y^4, x^2 + 2zy, 2z^2x^3 - y^3, 4z^3x + y^2, 2yx + z^4,$ $5x\partial_x + 7y\partial_y + 3z\partial_z + 36$
cohomology	-36	$\frac{-2}{17}\frac{1}{xyz^8} + \frac{-4}{17}\frac{1}{xy^4z} + \frac{1}{17}\frac{1}{x^2y^2z^4} + \frac{-2}{17}\frac{1}{x^4yz^3}$

S_{17} (4,10,7)	24	$f = x^6 + xy^2 + yz^2$
f_x	21	$f_1 = 6x^5 + y^2$
f_y	14	$f_2 = 2yx + z^2$
f_z	17	$f_3 = 2zy$
Ann		$x^6, 6x^5 + y^2, zx^5, zy, 2yx + z^2, 4x\partial_x + 10y\partial_y + 7z\partial_z + 51$
cohomology	-51	$\frac{-1}{4}\frac{1}{xy^4z} + \frac{-1}{12}\frac{1}{x^5yz^3} + \frac{1}{24}\frac{1}{x^6y^2z}$

U_{14} (3,3,2)	9	$f = x^3 + y^3 + yz^3$
f_x	6	$f_1 = 3x^2$
f_y	6	$f_2 = 3y^2 + z^3$
f_z	7	$f_3 = 3z^2y$
Ann		$x^2, y^3, z^2y, 3y^2 + z^3, x\partial_x + 2, 3y\partial_y + 2z\partial_z + 13$
cohomology	-19	$\frac{-1}{9}\frac{1}{x^2yz^5} + \frac{1}{27}\frac{1}{x^2y^3z^2}$

N_{19} (12,4,5)	24	$f = x^2 + y^6 + yz^4$
f_x	12	$f_1 = 2x$
f_y	20	$f_2 = 6y^5 + z^4$
f_z	19	$f_3 = 4z^3y$
Ann		$x, y^6, z^3y, 6y^5 + z^4, 4y\partial_y + 5z\partial_z + 39$
cohomology	-51	$\frac{-1}{8}\frac{1}{xyz^7} + \frac{1}{48}\frac{1}{xy^6z^3}$

${}_1N_{20}$ (19,8,6)	38	$f = x^2 + y^4z + yz^5$
f_x	19	$f_1 = 2x$
f_y	30	$f_2 = 4zy^3 + z^5$
f_z	32	$f_3 = y^4 + 5z^4y$
Ann		$x, y^7, zy^4, y^4 + 5z^4y, 4zy^3 + z^5, 4y\partial_y + 3z\partial_z + 31$
cohomology	-81	$\frac{-2}{19}\frac{1}{xyz^9} + \frac{1}{38}\frac{1}{xy^4z^5} + \frac{-5}{38}\frac{1}{xy^7z}$

N_{20} (25,10,8)	50	$f = x^2 + y^5 + yz^5$
f_x	25	$f_1 = 2x$
f_y	40	$f_2 = 5y^4 + z^5$
f_z	42	$f_3 = 5z^4y$
Ann		$x, y^5, z^4y, 5y^4 + z^5, 5y\partial_y + 4z\partial_z + 41$
cohomology	-107	$\frac{-1}{10}\frac{1}{xyz^9} + \frac{1}{50}\frac{1}{xy^5z^4}$

V_{15} (2,2,3)	8	$f = x^4 + y^4 + yz^2$
f_x	6	$f_1 = 4x^3$
f_y	6	$f_2 = 4y^3 + z^2$
f_z	5	$f_3 = 2zy$
$\mathcal{A}nn$		$x^3, y^4, zy, 4y^3 + x^2, x\partial_x + 3, 2y\partial_y + 3z\partial_z + 11$
cohomology	-17	$\frac{-1}{8}\frac{1}{x^3yz^3} + \frac{1}{32}\frac{1}{x^3y^4z}$

${}_1V^*_{18}$ (5,4,8)	20	$f = x^4 + y^5 + yz^2$
f_x	15	$f_1 = 4x^3$
f_y	16	$f_2 = 5y^4 + z^2$
f_z	12	$f_3 = 2zy$
$\mathcal{A}nn$		$x^3, y^5, zy, 5y^4 + z^2, y\partial_y + 2z\partial_z + 7, x\partial_x + 3$
cohomology	-43	$\frac{-1}{8}\frac{1}{x^3yz^3} + \frac{1}{40}\frac{1}{x^3y^5z}$

${}_2V^*_{18}$ (7,5,4)	19	$f = x^2y + y^3z + z^3x$
f_x	12	$f_1 = 2yx + z^3$
f_y	14	$f_2 = x^2 + 3zy^2$
f_z	15	$f_3 = 3z^2x + y^3$
$\mathcal{A}nn$		$x^4, yx^2, y^3x, y^6, -zx^3 + y^5, x^2 + 3zy^2, 3z^2x + y^3, 2yx + z^3,$ $7x\partial_x + 5y\partial_y + 4z\partial_z + 41$
cohomology	-41	$\frac{-2}{19}\frac{1}{xy^2z^6} + \frac{-3}{19}\frac{1}{xy^6z} + \frac{1}{19}\frac{1}{x^2y^3z^3} + \frac{-3}{19}\frac{1}{x^4yz^2}$

${}_1V^*_{19}$ (8,6,13)	42	$f = x^4 + xy^4 + yz^2$
f_x	24	$f_1 = 4x^3 + y^4$
f_y	26	$f_2 = 4y^3x + z^2$
f_z	19	$f_3 = 2zy$
$\mathcal{A}nn$		$-x^4, 4x^3 + y^4, -zx^3, zy, 4y^3x + z^2, 8x\partial_x + 6y\partial_y + 13z\partial_z + 69$
cohomology	-69	$\frac{-1}{8}\frac{1}{xy^8z} + \frac{-1}{8}\frac{1}{x^3yz^3} + \frac{1}{32}\frac{1}{x^4y^4z}$

${}_2V^*_{19}$ (6,8,11)	30	$f = x^5 + xy^3 + yz^2$
f_x	24	$f_1 = 5x^4 + y^3$
f_y	22	$f_2 = 3y^2x + z^2$
f_z	19	$f_3 = 2zy$
$\mathcal{A}nn$		$x^5, 5x^4 + y^3, zx^4, zy, 3y^2x + z^2, 6x\partial_x + 8y\partial_y + 11z\partial_z + 65$
cohomology	-65	$\frac{-1}{6}\frac{1}{xy^6z} + \frac{-1}{10}\frac{1}{x^4yz^3} + \frac{1}{30}\frac{1}{x^5y^3z}$

${}_3V^*_{19}$ (6,9,5)	20	$f = x^4 + xy^2 + yz^3$
f_x	18	$f_1 = 4x^3 + y^2$
f_y	15	$f_2 = 2yx + z^3$
f_z	19	$f_3 = 3z^2y$
$\mathcal{A}nn$		$x^4, 4x^3 + y^2, z^2x^3, z^2y, 2yx + z^3, 6x\partial_x + 9y\partial_y + 5z\partial_z + 52$
cohomology	-52	$\frac{-1}{6}\frac{1}{xy^4z^2} + \frac{-1}{12}\frac{1}{x^3yz^5} + \frac{1}{24}\frac{1}{x^4y^2z^2}$

V^*_{20} (8,10,15)	40	$f = x^5 + y^4 + yz^2$
f_x	32	$f_1 = 5x^4$
f_y	30	$f_2 = 4y^3 + z^2$
f_z	25	$f_3 = 2zy$
$\mathcal{A}nn$		$x^4, y^4, zy, 4y^3 + z^2, x\partial_x + 4, 3z\partial_z + 2y\partial_y + 11$
cohomology	-87	$\frac{-1}{10}\frac{1}{x^4yz^3} + \frac{1}{40}\frac{1}{x^4y^4z}$

V'_{20}	$(7,6,9)$	21	$f = x^3 + xy^3 + yz^3$
f_x		14	$f_1 = 3x^2 + y^3$
f_y		27	$f_2 = 3y^2x + z^3$
f_z		24	$f_3 = 3z^2y$
Ann			$x^3, 3x^2 + y^3, z^2x^2, z^2y, 3y^2x + z^3, 7z\partial_z + 6y\partial_y + 9x\partial_x + 59$
cohomology		-59	$\frac{-1}{9}\frac{1}{xy^6z^2} + \frac{-1}{9}\frac{1}{x^2yz^5} + \frac{1}{27}\frac{1}{x^3y^3z^2}$

V'_{21}	$(9,12,8)$	36	$f = x^4 + y^3 + yz^3$
f_x		27	$f_1 = 4x^3$
f_y		24	$f_2 = 3y^2 + z^3$
f_z		28	$f_3 = 3z^2y$
Ann			$x^3, y^3, z^2y, 3y^2 + z^3, x\partial_x + 3, 2z\partial_z + 3y\partial_y + 13$
cohomology		-79	$\frac{-1}{12}\frac{1}{x^3yz^5} + \frac{1}{36}\frac{1}{x^3y^3z^2}$

$(3(4n+k), 2(4n+k), 4)$	$6(4n+k)$	$f = x^2 + y^3 + yz^{4n+k}$, $n \geq 1$, $k = 1, 3$
f_x	$6(4n+k)$	$f_1 = 2x$
f_y	$4(4n+k)$	$f_2 = 3y^2 + z^{4n+k}$
f_z	$6(4n+k) - 4$	$f_3 = (4n+k)z^{4n+k-1}y$
Ann		$x, y^3, z^{4n+k-1}y, 3y^2 + z^{4n+k}$, $(4n+k)y\partial_y + 2z\partial_z + 20n + 5k - 2$
cohomology	$-(13(4n+k) - 4)$	$\frac{-1}{2(4n+k)}\frac{1}{xyz^{2(4n+k)-1}} + \frac{1}{6(4n+k)}\frac{1}{xy^3z^{4n+k-1}}$

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