Construction of a Model for the FeynArts and its Application to Particle Physics

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Abstract

We construct a model for a mathematical package the *FeynArts* programmed on the *Mathematica.* The model allows us to compute Feynman amplitudes for the hadronic system in a symbolic manner on computers. We apply the model to hadron physics. The pion photoproduction reaction on the nucleon $\gamma N \rightarrow \pi N$ and its radiative reaction $\gamma N \rightarrow \gamma\pi N$ are studied with the model code.

1. Introduction

In high energy particle physics, the symbolic computations on computers have a long history since the *REDUCE* was invented. The code has been extensively used for the trace calculation of Feynman amplitudes in Quantum Electrodynamics (QED). Recently more advanced and sophisticated tools have been available for symbolic manupilations of mathematical formulas on computers, for example the *Mathematica*(1). They have been widely used not only in pure and/or applied mathematics but also in many other fields in sciences and technologies. In this circumstance, a mathematical package called the *FeynArts*(2) has been developed for a particular interest in performing the symbolic manipulations of the Feynman amplitudes in particle physics. The package is executable on the *Mathematica.* The *FeynArts* has been used in drawing Feynman diagrams and calculating Feynman amplitudes for QED and Quantum Chromonodynamics (QCD). So far the package is available only for the renormalizable field theories such as QED and QCD, where one can calculate higher order loop diagrams as desired accuracy.

In this note we have developed a model program for calculating the Feynman ampli-

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tudes of the hadronic system such as photons (γ), mesons (π, ρ, ω) and nucleons (N). However the follwoing remark has to be noted in using the model. The strong interaction of the hadronic system is NOT renormalizable, therefore the use of the code should be restricted to the tree diagrams (not to the loop diagrams). We have also applied the model program to particle physics problems.

A brief introduction of the *FeynArts* is given in section 2. We construct a model program for the hadronic system there. The application of the code to particle physics is demonstarated in section 3. We study two examples: (1) the pion photoproduction reaction on the nucleon $\gamma N \rightarrow \pi N$ and (2) its radiative reaction $\gamma N \rightarrow \gamma\pi N$.

2. FeynArts and Model Construction

The *FeynArts* is one of the mathematical packages on the *Mathematica* which calculates the Feynman amplitudes in the symbolic manner for QED, QCD and other interaction models. The information in for the *FeynArts* is given in detail in Ref. 2. In this note, we concentrate in a model construction for the hadronic system. The model is essential in performing the *FeynArts* for the hadronic system. Again the following remark is in order. Unlike QED and QCD the model for the hadron system is unrenormalizable. Therefore the use of the model is restricted to the tree calculation of the Feynman diagrams.

Staring with the standard pseudovector Lagrangians(3), we find the following propagator and vertex expressions for photons, pions and nucleons.

Nucleon propagator:

$$\frac{i(\gamma_\mu p^\mu + m_N)}{p^2 - m_N^2}, \tag{1}$$

where m_N and p are the mass and the momentum of the nucleon, respectively. The Greek letter γ_μ denotes the Dirac gamma matrix.

Pion propagator:

$$\frac{i}{k^2 - m_\pi^2}, \tag{2}$$

where m_π and k are the mass and the momentum of the pion.

πNN vertex:

$$-i\,\frac{f_{\pi NN}}{m_\pi}\,\gamma_5\,\gamma_\mu\,\tau_i\,p^\mu, \tag{3}$$

where $f_{\pi NN}$ is the πNN coupling constant, and τ_i is the isospin Pauli matrix.

γNN vertex:

$$-e\left(\frac{1+\tau_3}{2}\right)\gamma_\mu, \tag{4}$$

where e is the electric charge of the proton.

$\gamma\pi\pi$ vertex:

$$-ie\,(k_\mu+k'_\mu)\,\varepsilon_{3ik}. \tag{5}$$

$\gamma\pi NN$ vertex:

$$-e\,\frac{f_{\pi NN}}{m_\pi}\,\gamma_5\,\gamma_\mu\,\tau_i\,\varepsilon_{3ik}. \tag{6}$$

$\gamma\gamma\pi\pi$ vertex:

$$2e^2\,g_{\mu\nu}\,(\delta_{ij}-\delta_{i3}\,\delta_{j3}). \tag{7}$$

Using eqs. (1)−(7), it is now straightforward to construct a model of the hadronic system for the *FeynArts*. In List. 1, we show the list of the obtained model program

List. 1

```
(* ************************************************************** *)
(*                                                                *)
(*                           model.PIN                            *)
(*                                                                *)
(* ************************************************************** *)

PropList[PIN]=
{
 Prop[in][ F[-13],F[13],mom_ ] ==
           PV[ NonCommutative[ DiracSlash[ mom ] + MN ]
               I PropagatorDenominator[ mom , MN ] ],
 Prop[ex][ F[13],mom_ ] == PV[ NonCommutative[ LeptonSpinor[ mom,MN] ] ],
 Prop[ex][ F[-13],mom_ ] == PV[ NonCommutative[ LeptonSpinor[ -mom,MN] ] ],
 Prop[in][ V[4,li1_],V[4,li2_],mom_] ==
          PV[    I  PropagatorDenominator[ mom , MLA ]
                   ( - MetricTensor[li1, li2]
                   + (1-1/GaugeXi[A]) FourVector[ mom, li1 ]
                     FourVector[ mom, li2 ]
                     PropagatorDenominator[ mom, MLA/Sqrt[GaugeXi[A]] ] ) ],
 Prop[ex][ V[4,li2_],mom_] ==
           PV[  PolarizationVector[ mom, li2 ] ],
 Prop[in][ S[4], S[4], mom_ ]    == PV[ I PropagatorDenominator[mom, MPI] ],
 Prop[ex][ S[4], mom_ ]     == PV[ 1 ]

}

                                 (** setting line specifications **)

LineSpec[ F[13] ]  = { straight, forward, "P" };
LineSpec[ F[-13] ] = { straight, backward, "P" };
LineSpec[ V[4] ]   = { wavy, none, Greek["g"] };
LineSpec[ S[4]   ] = { dashed, none, Greek["p"] }

CoupList[PIN]=
{
 Coup[{V[4,li1_],mom1_},{F[-13],mom2_},{F[13],mom3_}]  ==
   PV[ NonCommutative[DiracMatrix[li1]]  EL  ],
 Coup[ {V[4,li1_],mom1_}, {S[4],mom2_}, {S[4],mom3_} ]   ==
                     PV[ (FourVector[mom2, li1] - FourVector[mom3, li1])  EL ],
Coup[ {S[4],mom1_}, {F[-13],mom2_}, {F[13],mom3_} ]        ==
                     PV[ NonCommutative[ DiracSlash[mom1] . DiracMatrix[5] ]
                         FPI  ],
 Coup[ {V[4,li1_],mom1_}, {S[4],mom2_}, {F[-13],mom3_}, {F[13],mom4_} ] ==
                     PV[ NonCommutative[DiracMatrix[li1,5]] EL*FPI  ]
}
```

"*model.PIN*". In List. 1, propagators, line specifications and coupling forms are defined. F, V and S denote the nucleon (fermion), photon (vector particle) and pion, respectively.

3. Application of the Model to Hadron Physics

3.1 Pion Photoproduction Reaction $\gamma N \rightarrow \pi N$

Let us first apply the constructed model to the pion photoproduction on the nucleon: $\gamma N \rightarrow \pi N$. Running the *FeynArts* with the model program "*model.PIN*", one obtains the Feynman diagrams shown in Fig. 1. The diagrams consists from four topologically distinct diagrams (Top. 1–Top. 4). The lines with arrows correspond to the nucleon fields. The wavy and dotted lines are the photon and pion fields, respectively.

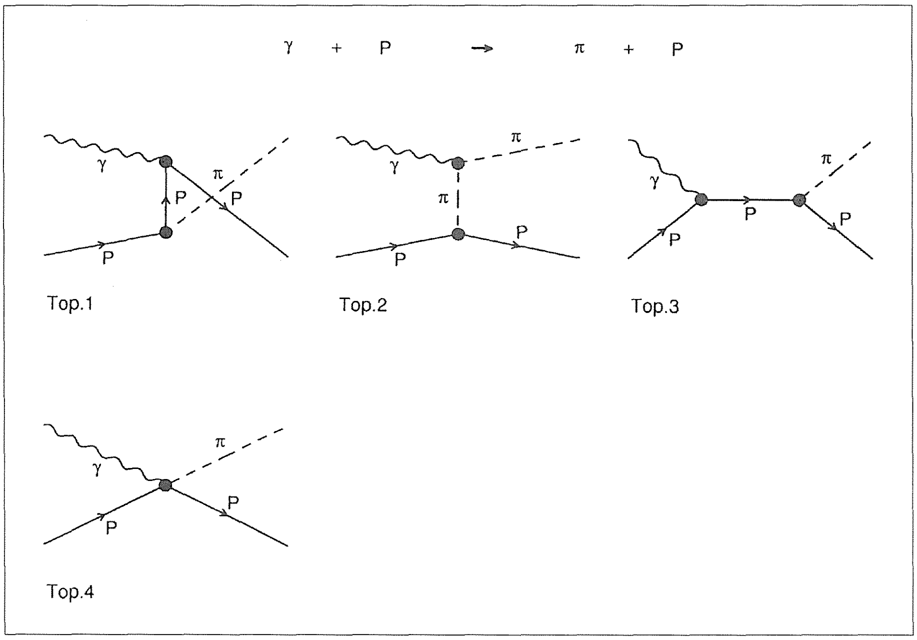

Fig. 1

The output of the calculated amplitudes is shown in List. 2. In List. 2 there exist four terms, which correspond to Top. 1 to Top. 4.

Let us write the explicit forms as follows.

Top. 1

$$e f_{\pi NN} \bar{u}_N(k_2) \gamma \cdot \varepsilon(p_1) \frac{\{\gamma \cdot (p_2 - k_1) + m_N\}}{(p_2 - k_1)^2 - m_N^2} \gamma \cdot k_1 \gamma_5 u_N(p_2), \tag{8}$$

List. 2

```
FeynAmpList[Model -> {PIN}, Generation1 -> True, Generation2 -> True,

>    Generation3 -> True, ElectronHCoupling -> True, LightFHCoupling -> True,

>    QuarkMixing -> False, UnitaryGauge -> False, RemoveEmptyTops -> True,

>    ProcessName -> V4F13S4F13,

>    Process ->

>     {{V[4], p1, 0}, {F[13], p2, MN}} -> {{S[4], k1, MPI}, {F[13], k2, MN}}]

>    [FeynAmp[V4F13S4F13, T1, I1, N1][-((EL FPI ep[p1, li2] Integral[]

>         u[k2, MN] . ga[li2] . (MN + gs[-k1 + p2]) . gs[-k1] . ga[5] .

                            2            2
>          u[p2, MN]) / (-MN  + (k1 - p2) ))],

>   FeynAmp[V4F13S4F13, T2, I1, N2][-((EL FPI ep[p1, li2] Integral[]

>         u[k2, MN] . gs[-k1 + p1] . ga[5] . u[p2, MN] (-k1)[li2]) /

             2            2
>       (-MPI  + (k1 - p1) ))],

>   FeynAmp[V4F13S4F13, T3, I1, N3][-((EL FPI ep[p1, li2] Integral[]

>         u[k2, MN] . gs[-k1] . ga[5] . (MN + gs[p1 + p2]) . ga[li2] .

                            2            2
>          u[p2, MN]) / (-MN  + (p1 + p2) ))],

>   FeynAmp[V4F13S4F13, T4, I1, N4][I EL FPI ep[p1, li2] Integral[]

>     u[k2, MN] . DiracMatrix[li2, 5] . u[p2, MN]]]
```

where $\bar{u}(k_2)$ and $u_N(p_2)$ are the nucleon spinors. The Greek letter $\varepsilon(p_1)$ denotes the photon polarization vector.

Top. 2

$$ef_{\pi NN}\bar{u}_N(k_2)\frac{\gamma\cdot(p_1-k_1)\gamma_5}{(p_1-k_1)^2-m_\pi^2}\gamma\cdot k_1\gamma_5 k_1\cdot\varepsilon(p_1)u_N(p_2), \tag{9}$$

Top. 3

$$ef_{\pi NN}\bar{u}_N(k_2)\gamma\cdot k_1\gamma_5\frac{\{\gamma\cdot(p_1+p_2)+m_N\}}{(p_1+p_2)^2-m_N^2}\gamma\cdot\varepsilon(p_1)u_N(p_2). \tag{10}$$

Top. 4

$$ief_{\pi NN}\bar{u}_N(k_2)\gamma\cdot\varepsilon(p_1)\gamma_5 u_N(p_2). \tag{11}$$

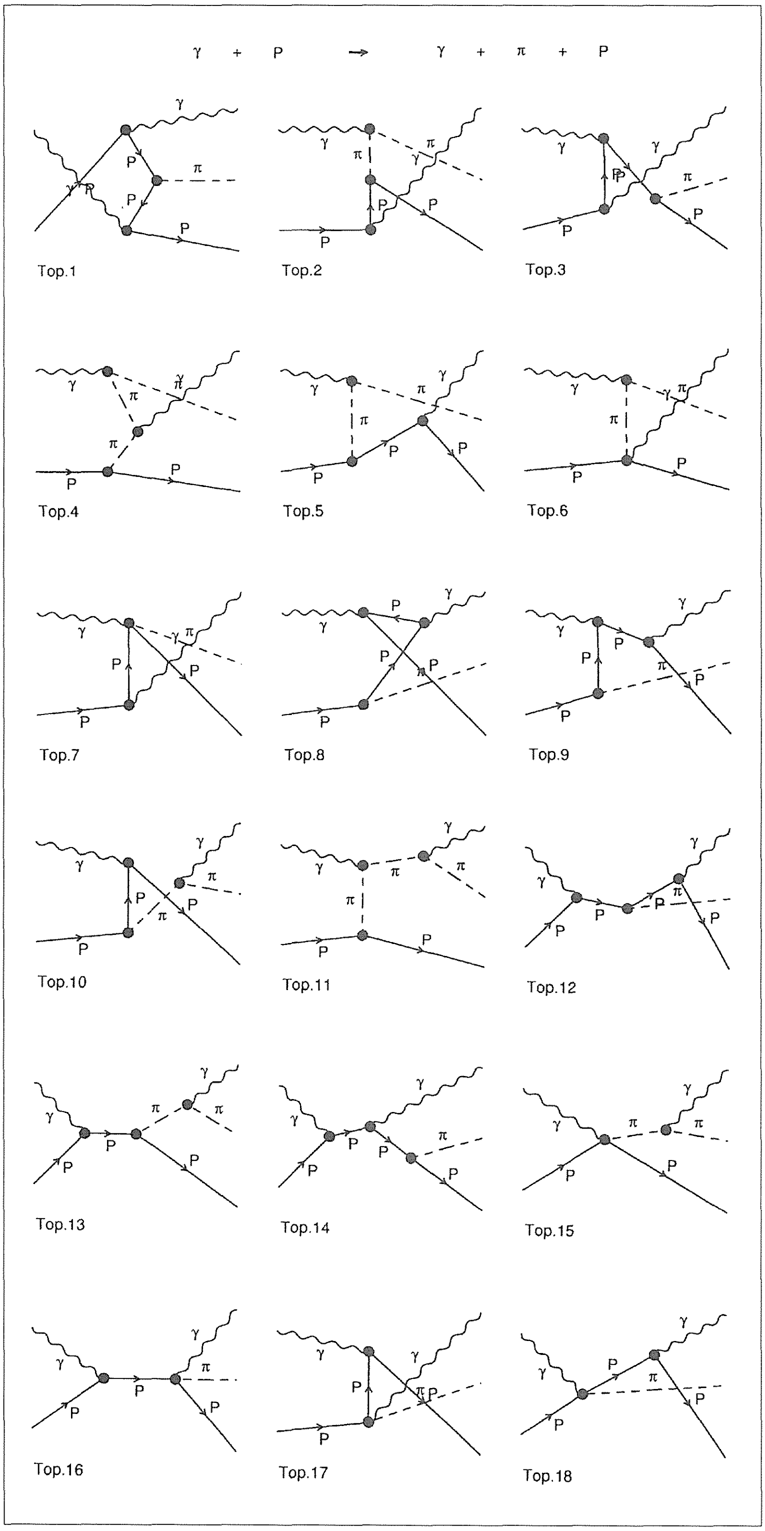
γ + P → γ + π + P
Top.1
Top.2
Top.3
Top.4
Top.5
Top.6
Top.7
Top.8
Top.9
Top.10
Top.11
Top.12
Top.13
Top.14
Top.15
Top.16
Top.17
Top.18
γ
P
π

Fig. 2

Equations (8) to (11) are the Feynman amplitudes for the reaction. All physics observables such as cross sections and polarizations are calculable with these amplitudes.

3.2 Radiative Pion Photoproduction Reaction $\gamma N \rightarrow \gamma\pi N$

The second example is $\gamma N \rightarrow \gamma\pi N$. In Fig. 2, we show the obtained Feynman diagrams. Top. 1 to Top. 16 are all topologically distinct diagrams. Similarly to the first example, one obtains sixteen Feynman amplitudes which correspond to Top. 1 to Top. 16. We skip showing the explicit forms in this note since it is rather lengthy. Instead we refer the recent articles by C. Wolfe et al.(4)(5). In Refs. 4 and 5, the physics implications have been explored in great detail.

In summary we have constructed a model of the hadronic system for the *FeynArts*. With the code one can perform symbolic computations for the hadronic system on the *Mathematica* program on computers. We have examined the model program for the physics applications. Two examples have been studied with the model: the pion photoproduction on the nucleon and its radiative reaction. The code is very useful and is essential in doing the symbolic manupilations of the Feynman amplitudes for the hadronic system on the *FeynArts*.

References

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(3) S. Nozawa, T.-S. H. Lee and B. Blankleider, Nucl. Phys. **A513** (1990) 459.

(4) C. E. Wolfe, S. Nozawa, M. N. Butler, B. Castel, 1994, submitted to Phys. Rev. C.

(5) C. E. Wolfe, Master Thesis, Queen's University, 1993, unpublished.