Pion Electroproduction on ^{3}He and the Δ-component

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Abstract

We investigate the pion electroproduction reaction ^{3}He(e, $e'\pi$) to study the preformed Δ-component in the ^{3}He wave function. It is shown that the previous analysis based upon the isospin symmetry with the Δ-dominance is not valid. It is argued that there is a considerable contribution from the Born diagrams. This implies that the large enhancement of the Δ knock-out process in the π^+/π^- ratio measurement may not be expected.

I. Introduction

It has been well known that the nucleus consists of not only nucleons but also other degrees of freedom such as mesons and excited baryons. Therefore the nuclear wave function has a certain probability of the excited baryons. It is a very fundamental and important question to ask whether such excited baryonic component really exists or not. Indeed, study of the preformed excited baryonic component in the nuclear wave function has a long history, in particular, for the Δ(1236) excitation. See Ref.(1) for the summary of the theoretical work and Ref.(2) for the experimental work.

The deuteron excites two Δ's because of the spin-isospin selection rule, a process which is energetically hindered. Therefore three-nucleon system such as ^{3}He is the simplest nucleus to study the preformed Δ-component. The Δ-component is generally very small. Therefore it is extremely difficult to identify the small quantity experimentally. Until recently, observables which are directly sensitive to the small component has not been well explored either experimentally nor theoretically.

Recently, Lipkin and Lee[(3)] have suggested that coincidence measurements of the

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charged pion electroproduction reaction ^{3}He(e, e') provide a clear test of the Δ-component. Using isospin symmetry properties of Δ, they found a large enhancement factor for detecting the Δ-component in measuring the π^+/π^- ratio in ^{3}He(e, $e'\pi$). The work triggered several proposals(4)(5) to determine the preformed Δ-component in ^{3}He with pion electroproduction measurements. Under such circumstance, more realistic calculations beyond the symmetry argument have been highly desired to cooperate with these forthcoming experiments.

One of the main objectives of the present project is to study various coincidence cross sections of ^{3}He(e, $e'\pi$) in the Δ-resonance energy region and to explore the sensitivity to the Δ-component in ^{3}He wave function.

II. Lipkin-Lee's prediction

Let us here review the Lipkin-Lee's argument for the enhancement in detecting the preformed Δ-component. They made the following assumptions.

(1) The Δ particle is an isospin 3/2 object.

(2) The strong interaction conserves the isospin.

(3) The $\gamma_{\Delta\Delta}$ coupling is proportional to the charge of Δ.

Using these assumptions, one obtains the relative magnitudes of the Δ photoproduction probability from single nucleon (denoted by the subscript n) in the following manner. The photoproduction on proton gives Δ^+, whereas neutron gives Δ^0. On the other hand, ^{3}He contains two protons and one neutron. This implies

$$P_n(\gamma^3\mathrm{He}\to\Delta^+X) : P_n(\gamma^3\mathrm{He}\to\Delta^0X) = \frac{2}{3} : \frac{1}{3} \tag{1.a}$$

and

$$P_n(\gamma^3\mathrm{He}\to\Delta^{++}X) = P_n(\gamma^3\mathrm{He}\to\Delta^-X) = 0. \tag{1.b}$$

With the help of isospin Clebsch-Gordan coefficients for $\Delta\to\pi N$, one obtains as follows.

$$P_n(\gamma^3\mathrm{He}\to\pi^+X) : P_n(\gamma^3\mathrm{He}\to\pi^0X) : P_n(\gamma^3\mathrm{He}\to\pi^-X) = \frac{2}{9} : \frac{6}{9} : \frac{1}{9} \tag{2}$$

Second, the relative Δ knock-out probabilities (denoted by the subscript k) are also calculated in a similar fashion.

$$P_k(\gamma^3\mathrm{He}\to\Delta^{++}X) : P_k(\gamma^3\mathrm{He}\to\Delta^+X) = \frac{6}{7} : \frac{1}{7} \tag{3.a}$$

and

$$P_k(\gamma^3\text{He}\to\Delta^0X)=P_k(\gamma^3\text{He}\to\Delta^-X)=0. \qquad (3.b)$$

Therefore

$$P_k(\gamma^3\text{He}\to\pi^+X):P_k(\gamma^3\text{He}\to\pi^0X):P_k(\gamma^3\text{He}\to\pi^-X)=\frac{19}{21}:\frac{2}{21}:0 \qquad (4)$$

Note that the π^- production is zero in P_k.

We now demonstrate that the π^+/π^- ratio is a good candidate to determine the preformed Δ-component. Let us start with the following hamiltonian.

$$H=H_0+V_{N\Delta}+V_{em}, \qquad (5)$$

where H_0 is the full hamiltonian in the nucleon space, and $V_{N\Delta}$ and V_{em} are the $N\leftrightarrow\Delta$ transition potential and the electromagnetic potential, which are treated perturbatively. With this hamiltonian the pion photoproduction amplitudes are given as follows. For the photoproduction from a single nucleon

$$\langle\gamma^3\text{He}|T_n|\pi^iX\rangle=\langle\gamma^3\text{He}|V_{em}|\Delta_qNN\rangle\langle\Delta_q|V_{N\Delta q}|\pi^iX\rangle\frac{1}{E-\tilde{E}_\Delta} \qquad (6)$$

and for the photoproduction from the preformed Δ-component

$$\langle\gamma^3\text{He}|T_k|\pi^iX\rangle=A_q\langle\Delta_q|V_{em}|\Delta_qNN\rangle\langle\Delta_q|V_{N\Delta q}|\pi^iX\rangle\frac{1}{E-\tilde{E}_\Delta} \qquad (7)$$

The relative magnitude Y of these cross sections is

$$Y=\frac{\sigma_k(\gamma^3\text{He}\to\Delta NN)}{\sigma_n(\gamma^3\text{He}\to\Delta NN)}=\frac{7}{9}P_\Delta\frac{|\langle|\Delta^+V_{em}|\Delta^+\rangle|^2}{|\langle p|V_{em}|\Delta^+\rangle|^2}, \qquad (8)$$

where

$$P_\Delta=\sum_q|A_q|^2 \qquad (9)$$

is the probability of finding Δ in ³He. Finally the π^+/π^- production ratio is

$$\frac{\sigma(\pi^+)}{\sigma(\pi^-)}=\frac{P_n(\gamma^3\text{He}\to\pi^+X)\sigma_n(\gamma^3\text{He}\to\Delta NN)+P_k(\gamma^3\text{He}\to\pi^+X)\sigma_k(\gamma^3\text{He}\to\Delta NN)}{P_n(\gamma^3\text{He}\to\pi^-X)\sigma_n(\gamma^3\text{He}\to\Delta NN)+P_k(\gamma^3\text{He}\to\pi^-X)\sigma_k(\gamma^3\text{He}\to\Delta NN)}=\left(2+\frac{57}{7}Y\right). \qquad (10)$$

It is straightforward to see that eq.(10)is sensitive to the Δ probability P_Δ because of the large enhancement factor. This the argument shown by Lipkin and Lee[(3)].

However, several questions still remain. Namely, the following two assumptions were made implicitly in the above discussion. They are as follows.

(4) Quasi-free kinematics (impulse approximation).

(5) There is no Born (background) pion photoproduction amplitude.

first, one has to check whether the quasi-free approximation is reasonable. If not two-body mechanism such as meson exchange current corrections have to be taken into account. More importantly, the Born contribution to the pion photoproduction was neglected completely in the derivation of eq. (10). In the case of the pion photo- and electroproduction on the nucleon $N(e, e'\pi)$, it is known[(6)(7)] that the Born contribution is sizable compared with the Δ-excitation even in the Δ energy region. Given these in mind, it is extremely important to check whether the assumptions (4) and (5) are suitable. In order to do this, one needs a dynamical model calculation for the pion photoproduction cross sections, which is one of our main purposes of the present project.

III. Dynamical Calculation of the Pion Electroproduction (Part 1)

Very recently Laget[(8)] made a calculation for ^{3}He$(e, e'\pi^+)$ including the Born contribution. He has demonstrated that indeed the Born diagram contribution is sizable compared with the Δ-excitation contribution. He has concluded that the large enhancement

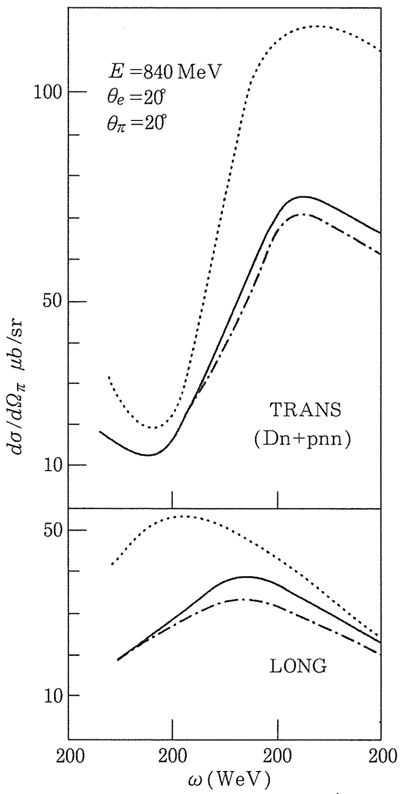

Fig. 1 The ^{3}He$(e, e'\pi)$ cross section[(8)].

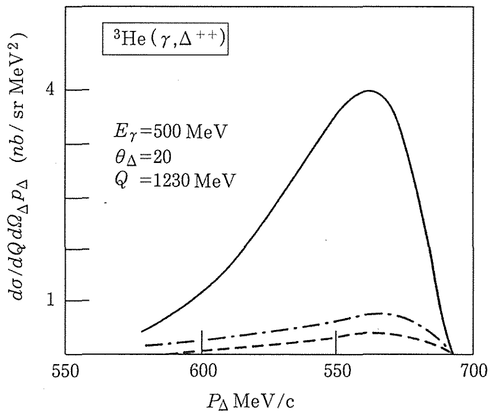

Fig. 2 The ^{3}He(γ, Δ^{++}) cross section[8].

of the preformed Δ is not expected due to the large background contribution of the Born diagrams. In Fig. 1, we have shown a figure taken from Ref. (8). The top figure shows the transverse cross section, where the Δ-component effect is negligible. The bottom figure presents the longitudinal cross section, where the Δ-component effect is small ~ a few %. Born term and meson exchange current contributions are also important.

Laget also suggested in Ref. (8) that the ^{3}He(γ, Δ^{++}) and ^{3}He(e, $e'\Delta^{++}$) cross sections will be sensitive to the preformed Δ-component. As demonstrated in eqs. (1) and (2), there is no Δ^{++} production contribution from the quasi-free nucleons in ^{3}He. The Δ^{++} production should come either from the preformed Δ-component or from meson exchange diagrams. In Fig. 2 we have shown the cross section for ^{3}He(γ, Δ^{++}) for illustrative purpose.

IV. Dynamical Pion Electroproduction (Part 2)

In the previous discussion, authors were always considering the observables which involve the Δ-probability P_Δ, which has been suggested $P_\Delta \simeq 1\%$. Although the probability $P_\Delta = |\epsilon_\Delta|^2 \simeq 0.01$ is small, the amplitude ϵ_Δ may not be necessarily small, i.e. $\epsilon_\Delta \simeq 0.1$ (10%). Therefore one could get 20% effect by extracting the interference contribution between the leading term and the preformed Δ term. We here suggest exclusive measurements of the ^{3}He(e, e') reaction in the off-scattering plane. The kinematics is shown in Fig. 3.

Let us introduce the ^{3}He wave function as follows.

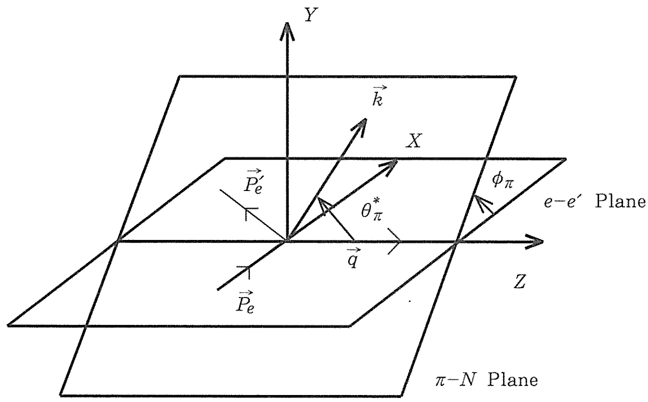

Fig. 3 Kinematics for ^{3}He$(e, e'\pi)$.

$$|\vec{p}, \beta> = \sqrt{1-\epsilon_\Delta^2}\,|\vec{p}, \beta; 3M> + \epsilon_\Delta |\vec{p}, \beta; \Delta, 2N>, \tag{11}$$

where the first and second terms are wave functions of the 3-nucleon space and the Δ-component. After the standard calculation, the cross section form for ^{3}He$(e, e'\pi)$ is finally obtained as follows.

$$\frac{d^3\sigma}{dE_e'\, d\Omega_e'\, d\Omega_\pi} = \frac{\alpha^2}{Q^2}\frac{E_e'}{E_e}\frac{1}{1-\epsilon}\left\{\frac{d\sigma_T'}{d\Omega_\pi} + \epsilon\frac{d\sigma_T'}{d\Omega_\pi} + \epsilon\cos 2\phi_\pi \frac{d\sigma_{TT}'}{d\Omega_\pi} + \sqrt{2\epsilon(1+\epsilon)}\cos\phi_\pi \frac{d\sigma_{TL}'}{d\Omega_\pi}\right\}. \tag{12}$$

Transverse cross section is given as follows.

$$\frac{d\sigma_T'}{d\Omega_\pi} = \frac{d\sigma_T}{d\Omega_\pi} + 2\epsilon_\Delta\sqrt{1-\epsilon_\Delta}\,\frac{d\sigma_T^{\text{interf}}}{d\Omega_\pi}, \tag{13}$$

where

$$\frac{d\sigma_T}{d\Omega_\pi} = \int d^3p_N \int_{-E_A+E_{A:}}^{\infty} dE S_A(\vec{p}_N, E)\{W^{xx}(\hat{k}, q_N, p_N) + W^{yy}(\hat{k}, q_N, p_N)\}_{\phi_\pi=0} \tag{14}$$

is the transverse cross section from the single nucleon in ^{3}He. The interference cross section is

$$\frac{d\sigma_T^{\text{interf}}}{d\Omega_\pi} = \text{Re}\left[\int d^3p_N \int_{-E_A+E_{A:}}^{\infty} dE S_A^{\text{interf}}(\vec{p}_N, E)\{W_{\text{interf}}^{xx}(\hat{k}, q_N, p_N) + W_{\text{interf}}^{yy}(\hat{k}, q_N, p_N)\}_{\phi_\pi=0}\right], \tag{15}$$

where $S_A(\vec{p}_N, E)$ is the spectral function for ^{3}He, and $S_A^{\text{interf}}(\vec{p}_N, E)$ is the interference spectral function.

V. Numerical Results

In the mean time, we have calculated the transverse and longitudinal cross sections (namely $d\sigma_T/d\Omega_\pi$ and $d\sigma_L/d\Omega_\pi$ to get the sensitivity of the order of $P_\Delta = |\epsilon_\Delta|^2$. For simplicity, we have used the most simple wave functions for the nucleon and Δ in the ^{3}He nucleus. The explicit form is given in Appendix A. In Fig. 4, the transverse and longitudinal cross sections are plotted for a kinematics: E_e = 645 MeV, ω = 290 MeV and θ_π = 90 deg. *The sensitivity of* P_Δ = 0.01 is shown in Figs. 5 and 6 for the transverse and longitudinal cross sections, respectively. As one can naively anticipate, the sensitivity were not seen in the transverse cross section and were very small for the longitudinal cross section. Therefore it is very important to consider the interference contribution which is of the order of $|\epsilon_\Delta|$ instead of P_Δ. The numerical calculation of this contribution is in progress.

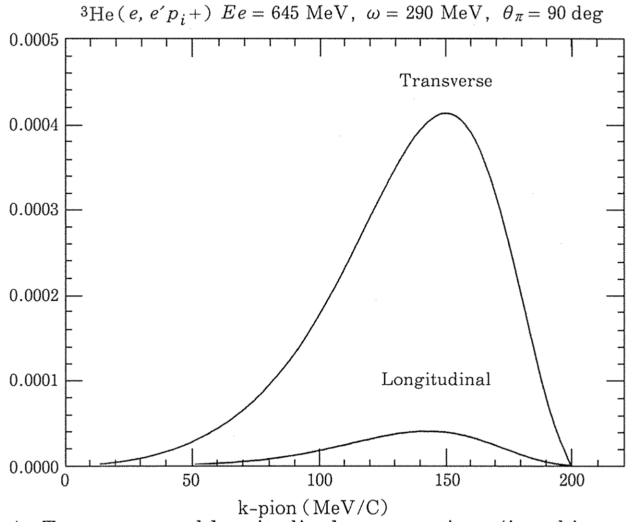

Fig. 4 Transverse and longitudinal cross sections (in arbitary units).

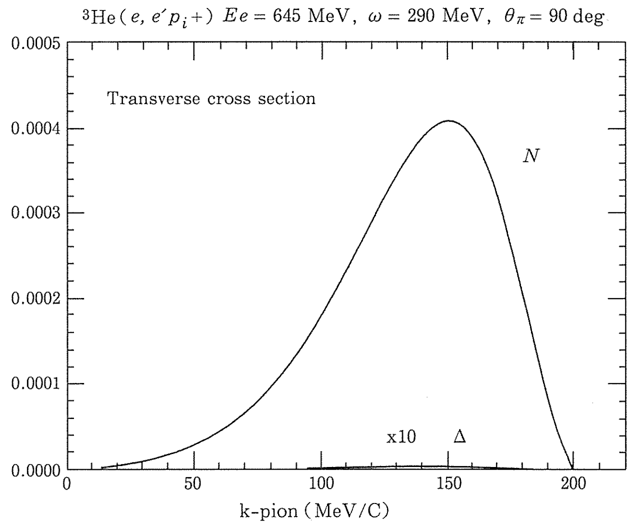

Fig. 5 Δ-component contribution in the transverse cross section.

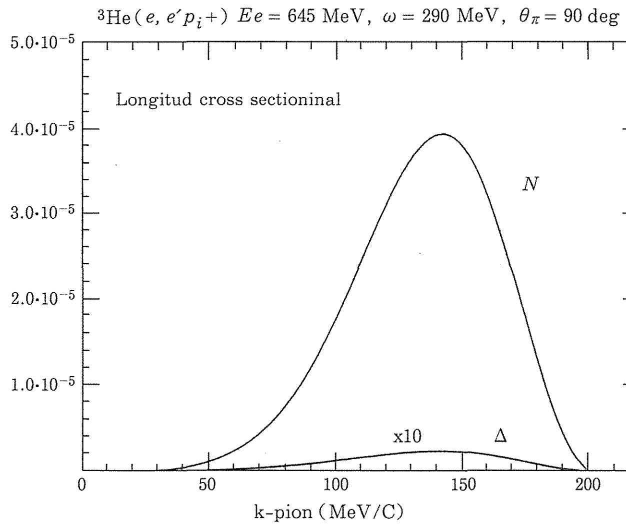

Fig. 6 Δ component contribution in the longitudinal cross section.

APPENDIX

A. The ^{3}He wave function

In general the ^{3}He wave function is given as follows.

$$|\Psi_{3He}\rangle=\sqrt{1-\epsilon_\Delta^2}\,|\Psi_{3N}\rangle+\epsilon_\Delta|\Psi_\Delta\rangle, \tag{A.1}$$

where ϵ_Δ is the amplitude of the Δ component in the ^{3}He wave function, and is typically $\epsilon_\Delta^2 = 0.01$. The first and second terms represent wave functions for the 3-nucleon component and the (Δ, 2N) component, respectively.

Let us first discuss the 3-nucleon wave function Ψ_{3N}. As shown in Fig. 1a, we introduce the following relative momenta

$$\vec{q}_i=\frac{1}{2}(\vec{p}_j-\vec{p}_k), \qquad \vec{K}_i=\frac{2}{3}\vec{p}_i-\frac{1}{3}(\vec{p}_j+\vec{p}_k), \tag{A.2}$$

where (i, j, k) is cyclic in (1, 2, 3), and the center of mass momentum

$$\vec{P}_A=\vec{p}_1+\vec{p}_2+\vec{p}_3. \tag{A.3}$$

In the present calculation, we approximate Ψ_{3N} by three Gaussians of the 0s state. For ^{3}He at rest ($\vec{P}_A=0$), we have the following form.

$$|\Psi_{3N}(\vec{P}_A=0)\rangle=|L=0, J=\frac{1}{2}, m_J;\, T=\frac{1}{2}, T_z\rangle\phi_{0s}(\vec{K}_1, \vec{q}_1) \tag{A.4}$$

The spacial wave function ($(0s)^3$) is symmetric and is given by

$$\phi_{0s}(\vec{K}_1, \vec{q}_1) = NY_{0,0}(\hat{K}_1)e^{-\frac{3}{4}b^2K_1^2}Y_{0,0}(\hat{q}_1)e^{-b^2q_1^2}, \quad \text{(A.5)}$$

where N is the normalization constant determined by

$$\int d^3K_1 d^3q_1 |\phi_{0s}(\vec{K}_1, \vec{q}_1)|^2 = 1. \quad \text{(A.6)}$$

The spin-isospin wave function is totally anti-symmetric and is

$$\left|L=0, J=\frac{1}{2}, m_J; T=\frac{1}{2}, T_z\right> = \left|\frac{1}{2}, \frac{1}{2}\frac{1}{2}(1); J=\frac{1}{2}, m_J\right>\left|\frac{1}{2}, \frac{1}{2}\frac{1}{2}(0); T=\frac{1}{2}, T_z\right> \quad \text{(A.7)}$$

where

$$|j_1, j_2j_3(j_{23}); J, J_z\rangle = \sum_{m_1, m_2, m_3, m_{23}} (j_1, m_1, j_{23}|J, J_z)(j_2, m_2, j_3, m_3|J_{23}, m_{23})$$
$$|j_1, m_1\rangle|j_2, m_2\rangle|J_3, m_3\rangle. \quad \text{(A.8)}$$

We then construct the (Δ, 2N) wave function Ψ_Δ. The kinematics is defined in Fig. 1b.

$$\vec{q}_1 = \frac{1}{2}(\vec{p}_2 - \vec{p}_3), \qquad \vec{K}_1 = \frac{2m_N}{m_\Delta + 2m_N}\vec{p}_1 - \frac{m_\Delta}{m_\Delta + 2m_N}(\vec{p}_2 + \vec{p}_3), \quad \text{(A.9)}$$

The center of mass momentum $\vec{P}_A$ is same as eq. (A.3). For ^{3}He at rest, the wave function is given explicitly as follows.

$$\left|\Psi_\Delta(\vec{P}=0)\right> = \left|\frac{3}{2}, \frac{1}{2}\frac{1}{2}(1); T=\frac{3}{2}, T_z\right> \sum_{m, m_\Delta, m_J} \frac{1}{\sqrt{2}}\left[\frac{1}{\sqrt{2}}\left|\frac{3}{2}, \frac{1}{2}\frac{1}{2}(0); J_\Delta=\frac{3}{2}, m_\Delta\right>\right.$$
$$\left.\{\phi_A(m_\Delta; \vec{K}_1, \vec{q}_1, + \phi_B(m_\Delta; \vec{K}_1, \vec{q}_1)\} + \left|\frac{3}{2}, \frac{1}{2}\frac{1}{2}(1); J_\Delta=\frac{1}{2}, m_\Delta\right>\phi_C(m_\Delta; \vec{K}_1, \vec{q}_1)\right] \quad \text{(A.10)}$$

The spacial wave functions ϕ_A, ϕ_B and ϕ_C are defined as follows.

$$\phi_A(m_\Delta; \vec{K}_1, \vec{q}_1) = N_AY_{2,m_\Delta}(\hat{K}_1)(bK_1)^2e^{-\frac{1}{4}(1+2\alpha)b^2K_1^2}Y_{0,0}(\hat{q}_1)e^{-b^2q_1^2} \quad \text{(A.11)}$$

$$\phi_B(m_\Delta; \vec{K}_1, \vec{q}_1) = N_BY_{0,0}(\hat{K}_1)e^{-\frac{1}{4}(1+2\alpha)b^2K_1^2}Y_{2,m_\Delta}(\hat{q}_1)(bq_1)^2e^{-b^2q_1^2} \quad \text{(A.12)}$$

$$\phi_C(m_\Delta; \vec{K}_1, \vec{q}_1) = N_C[Y_1(\hat{K}_1)\times Y_1(\hat{q}_1)]^{(2)}_{m_\Delta}(bK_1)e^{-\frac{1}{4}(1+2\alpha)b^2K_1^2}(bq_1)e^{-b^2q_1^2}, \quad \text{(A.13)}$$

where $\alpha = m_N/m_\Delta$. The relative angular momentum states for the wave functions are shown in Table 1.

Table 1 The (Δ, 2N) wave function.

wave function	$\vec{q}_1$	$\vec{K}_1$	$\vec{P}$
A	S	D	0
B	D	S	0
C	P	P	0

References

(1) A. M. Green, *Mesons in Nuclei,* page 277, edited by M. Rho and D. H. Wilkinson, North-Holland Publishing Company, 1979.

(2) L. S. Kisslinger, page 261, *ibid.*

(3) H. Lipkin and T.-S. H. Lee, Phys. Lett. **B183** (1987) 22.

(4) R. Milner and T. W. Donnelly, Phys. Rev. **C37** (1988) 870.

(5) M. Moinester, preprint.

(6) S. Nozawa and B. Blankleider and T.-S. H. Lee, Nucl. Phys. **A513,** 459 (1990) .

(7) S. Nozawa and T.-S. H. Lee, Nucl. Phys. **A513,** 511 (1990) and *ibid.* **A513,** 543 (1990) .

(8) J. M. Laget, Proceeding of the Workshop on Electronuclear Physics with Internal Targets and The Blast Detector, Arizona, March 1992.