

On a Recent Discussion of the Transformation Problem

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§ 1. Introduction.

1. It seems that the transformation problem, posed by Marx and criticised later by Bortkiewicz [07], has reached a widely acceptable solution in these years. This solution is based on a given vector of wage goods, and makes a full use of the well known Peron-Frobenius theorem in discussing the existence of the unique rate of profit and associated proportions of prices. Recently, however, Lipietz [82], being unhappy with the accepted one, spent another bottle of ink to tackle the transformation problem, and two economists made comments to his new solution.¹⁾

The first objective of this short comment is to review some formal background of his "new solution".²⁾

In the second place, we shall examine if Lipietz's way of thinking will bring us something novel or important. Since his argument seems to leave basic points unclear, it will be necessary to see what is the basis of his new solution.

We shall employ an ordinary Leontief economy framework with n types of commodities. We employ our own notation system :

- A $n \times n$: input coefficient matrix,
- L $1 \times n$: labour input vector,
- f $n \times 1$: wage good bundle vector,
- p $1 \times n$: production price vector,
- ω : real wage rate,

1) The two commentators are Dumenil and Flaschel.

2) This note is an extension of author's communication to A. Lipietz, which commented on an earlier draft. It must be noted that our stance may be very close to that of Flaschel [84] with regards to the formal aspect.

- π : rate of profit,
 x $n \times 1$: output proportion,
 y $n \times 1$: net product vector,
 w $1 \times n$: value vector,
 μ : the rate of surplus value (exploitation).

The first two, A and L , are regarded as given technical data: the third one, f , will be treated as given if introduced, whereas the remaining ones except the last four are main variables concerning the price system.³⁾

§2. The accepted theory of transformation.

1. As well known, starting from the value system, Marx attempted to establish the production price system. His formulation and its ground remained to be vague, so that the problem was left. The problem is to prove how prices and the rate of profit are dependent on and evaluated from the value system, with the subsequent two equalities:

- M1 total price = total value,
 M2 total profit = total surplus value.

Bortkiewicz paved the way for rigorous formulation of the problem, whereas Sraffa [60] presented a precise formulation of the production price system. The standardised transformation procedure accepted today is based on this stream.

Let us briefly review the accepted theory of transformation.

The production price system $(p, \pi, \omega) \in \mathbb{R}^n \times \mathbb{R} \times \mathbb{R}$ is the one to satisfy:

$$(1) \quad p = (1 + \pi)(pA + \omega L),$$

The equation system of production price (1) has n equations in $n+2$ variables, so that there is a degree of freedom 2 in this system.

There are various ways to close the system. The accepted transformation procedure, which Lipietz called the Morishima-type, usually closes this by giving the wage good vector.

Suppose that values of labour-power are determined by their reproduction cost: let the wage good bundle purchased by workers $f \in \mathbb{R}^n$ be given, and one has

$$(1) \quad p = (1 + \pi)(pA + \omega L),$$

$$(2) \quad \omega = pf,$$

These two can be put into one as:

$$(3) \quad p = (1 + \pi)pM,$$

3) As for details, see Fujimori [82].

where

$$M = A + fL.$$

Although this system is not closed, the rate of profit π is determined by $\frac{1}{\rho[M]} - 1$, and the ratio of production prices of commodities are determined as the eigenvector of M associated with $\rho[M]$, where $\rho[M]$ indicates the Frobenius root of M .

2. It must be observed that although proportions of production prices and the rate of profit can be evaluated by solving (3), there still remains one degree of freedom in the system of equations (3). If one introduces any single reasonable condition, then the absolute level of production prices will be evaluated. Needless to say, this may be either M1 or M2, if one wishes:

The value system $w \in \mathbb{R}^n$ is described by:

$$(4) \quad w = wA + L,$$

so that

$$(4') \quad w = L(I - A)^{-1}.$$

If we adopt M1, for instance, then we have (1) - (4) and

$$(5) \quad px = wx, \quad (x \in \mathbb{R}^n).$$

Now, equations (1) - (5), System A, constitute a complete closed system to determine the production price system. It must be remembered that Okishio [74] discussed an iteration procedure which is based on (5), and this procedure generalises exactly what Marx intended to complete through his numerical examples.

The core of the above story does not change much, even if we adopt M2 in lieu of M1.⁴⁾

It should be observed that in this accepted transformation procedure, no more equation, or no more normalisation can be made further. That is, two conditions, M1 and M2, are no longer fulfilled simultaneously except for some specific x . The proportion of outputs which permits both M1 and M2 is the well known von Neumann ray. It must be remarked that the accepted theory of transformation is a dual theory: it develops quantity equilibrium as well as price equilibrium.

As for the rate of surplus value, it is evaluated by

$$(6) \quad \mu = \frac{1}{wf} - 1$$

4) As for the iteration with M2, see Fujimori [82], p. 61.

and π and μ move to the same direction as each component of f increases or decreases. The fundamental Marxian theorem that π is positive if and only if μ is positive holds with the above defined μ .

§ 3. Lipietz's reinterpretation.

1. Lipietz [82] criticizes the above approach, by saying that the rate of profit is independent of the output proportion, and that value and surplus value are not conserved in the transformation. According to Lipietz [82], these deficits are due to mistreatment of the reproduction of labour-power, i. e., human beings are treated just like horses or cows.⁵⁾ He argues that such a view is not correct: this simplifies the complex, circular relationship among the value of labour-power, wage goods and distribution. He thus rejects the determination of the value of labour-power by the value of wage goods. Hence, in order to discuss exploitation he has to introduce some other relationships concerning the value of labour-power.

Then, he introduces the two well-known equalities as definitions. The first one is an identity—a distribution relationship in the zero profit economy:

$$v(1+\mu)=1.$$

Lipietz argues that M1 should be applied to the outcome of direct labour alone, and not to the gross product. He thus introduces the second definitional equality: the price of net products is equal to the amount of direct labour, i. e.,

$$py=Lx, \quad y=x-Ax.$$

So far, Lipietz's system can be summarised formally by:

- (1) $p=(1+\pi)(pA+\omega L),$
- (2) $py=Lx,$
- (3) $v(1+\mu)=1,$
- (4) $y=x-Ax.$

Since (4) is outside the price world, we see that his system has $n+2$ equations in $n+4$ variables, p, π, ω, v, μ . Even if one of variables, say μ , is given, the remaining variables are not all determined: we have to add one more. This is made clearer in the following.

From the above, Lipietz deduces the following theorem which is restated in our notation:

5) The same type of criticism is found in Sekine [84].

THEOREM. (Lipietz) Suppose that the value of labour-power is equal to real wages :

$$(5) \quad v = \omega.$$

Then :

- (i) π is unique.
- (ii) M2 holds: total profit = total surplus value.
- (iii) π is a function of A, L, μ and x .

In fact, (ii) is easy to see: the result soon follows in view of

$$(6) \quad w = wA + L = L(I - A)^{-1}.$$

(i) and (iii) are already established facts.

It should be observed that Lipietz's system becomes self-contained if supplemented by (5).

Remark also that from his system (1)-(5), or System B, the antagonistic relationship between real wages and the rate of surplus value follows immediately. The fundamental Marxian theorem follows also immediately.

2. One may consider, of course, another normalisation to close the system (1). Since Marx mentioned two equalities, i. e., M1 and M2, one may well be tempted to introduce such normalisations immediately.

This way to close the system will be as follows: (System C)

- (1) $p = (1 + \pi) (pA + \omega L),$
- (3) $v(1 + \mu) = 1,$
- (7) $px = wx,$
- (8) $\pi(pA + \omega L)x = v\mu Lx.$

with

$$(6) \quad w = wA + L = L(I - A)^{-1}.$$

Here, we have $n+3$ equations (1), (3), (7) and (8), in $n+4$ variables, p, π, ω, v and μ . If one of those variables, say v , is given, the remaining variables are all determined.

From the above, it soon follows that the value of capital is equal to price of capital. We can also infer that the rate of profit is unique as usual. Moreover, the rate of profit π can be represented by

$$\pi = \frac{v\mu Lx}{pAx + \omega Lx} = \frac{v\mu Lx}{wAx + vLx} = \frac{\mu}{\frac{wAx}{vLx} + 1}$$

This indicates that the rate of profit is the rate of surplus value divided by $1 +$ organic composition of capital. From the above formulæ, it soon follows that the rate of profit is an increasing function of the rate of surplus value. It also follows that in value terms the rate of profit is immediately evaluated. This feature does not follow from Lipietz's choice of normalisation.⁶⁾

§4. Concluding remarks.

1. Thus far, we have looked at the transformation Problem from rather a formal viewpoint. By way of conclusion, let us make some points further.

Lipietz [82] criticized the accepted approach with regards to the independence of the proportion of production prices from the proportion of outputs, but he only looked at one side of the accepted theory of transformation, and not its whole figure. We have to stress that the accepted one is a dual theory. The price equilibrium is always accompanied by the quantity equilibrium. In the form of duality, the accepted theory of transformation takes into consideration the proportion of outputs.

According to Lipietz's argument, the dependence of production prices on the proportion of outputs means that production prices should be established on any proportions of output. When Lipietz mentions this dependence of production prices on the proportion of outputs, he may have in mind the supply-demand equality in the market: he seems to assume tacitly that the production price should be the market equilibrium price. Those two price systems are, however, of different nature. The theory of transformation is not to discuss the transformation of values into equilibrium prices to which market prices converge.⁷⁾⁸⁾

As for the conservation of values and surplus values, we reviewed the problem from the standpoint of normalisation.

The essence of Lipietz's argument lies in his choice of normalisation based on the degree of freedom of the equation system. Different from the accepted transformation procedure, he introduced (2) and (5).

Since prices or values of commodities are divided into three components, i. e., constant capital, variable capital and surplus value or profits, one can consider various equalities of those components between value and price terms. Lipietz's choice is one option. Note that any two of (2), (5) and M2 imply the remaining one. It may not be fair to say that M2 alone can be a conclusion.

It must be observed that Lipietz's System B, i. e., (1)–(5), is closed, so that it

6) As for the relationship between the transformation and the normalisation, also refer to Fujimori [82], p. 19.

7) Some economists have discussed the historical transformation of values into prices, but they may confuse the relationship between values and production prices and the relationship between market prices and production prices.

8) It may be worth mentioning that a given wage good bundle represents an aspect of consumption demand from workers.

never allows additional normalisation in general. Hence, his assertion which he made in comparing his solution with the accepted one— “If the numeraire is chosen so that the sum of prices = the sum of values, ...” (Lipietz [82], p. 82)— may not be correct with his system.⁹⁾

2. An important point is how to grasp the nature of labour-power. None will deny that labour-power is commoditized in the capitalist economy. Marx made a clear distinction between labour and labour-power, maintained that labour-power is commoditized, and discussed how it will be reproduced as a commodity. The accepted theory of transformation seems to follow this tradition. Whether one likes it or not, one's labour-power is transacted in the labour market, as cows and horses in the commodity market. Even if individual workers have different tastes, preferences and so on, the average wage good bundle can be evaluated in the long run. Wages are, after all, a part of the advanced cost of capitalist production. In the light of this stream of reasoning, the accepted theory of transformation is a very straightforward, but natural treatment of the reproduction of labour-power.¹⁰⁾

As seen in the preceding section, Lipietz [82] rejects a given wage good bundle, a given wage good bundle leads, according to Lipietz, to treat variable capital in the same manner as constant capital in the sense that costs of inputs are merely evaluated. We have to point out that variable capital is called variable, because it can create a greater magnitude of value in production: even if the wage good bundle is given, its variableness does not change. Labour-power is a still specific type of commodity. It seems, however, that Lipietz gave a different meaning to “variable” capital.

We have constructed, in a manner similar to Lipietz's approach, System C, in which Marx's original equalities M1 and M2 are implemented.¹¹⁾ We have to say, however, that this means choosing another type of normalisation. This was not done to show the validity of his new solution, but, on the contrary, it suggests that an option of normalisation will bring various “equalities”, as we wish. Lipietz could have chosen such a normalisation as M2 from the very beginning.

3. With reference to the normalisation and the value of labour-power, it will be necessary to mention the relationship between (2) and the grasp of labour-power.

By comparing System B and System C, we see that the fundamental Marxian theorem is shown immediately, if M1 or M2 is combined with this definition. Straightforwardness in showing the theorem comes from the identity $v(1+\mu)=1$.

This identity means the following: unity in the right-hand side of the equality sign indicates that total net products are regarded as an outcome of labour, whereas v the ratio of net products which comes back to the hands of workers.

9) At least Lipietz has to show a specific x for which his assertion is valid.

10) We do not say that it is the best and final solution.

11) Rather, Lipietz's normalisation is a modification of Marx's original equalities.

We see that this identity pushes distribution ahead of the reproduction of labour-power.

Now, we can characterise Lipietz's approach. In contrast to the accepted theory of transformation, Lipietz's approach may be featured as distributional one, because the theory of transformation is, for Lipietz, to explain the distribution with a positive rate of profits in terms of the distribution with zero rate of profits.

Now, in the price world no unequal exchange takes place in the transaction of labour-power and wages often appear as distribution to workers. Without violating the law of commodity production, exploitation occurs: equal exchange, but exploitation, because the price world obscures the distinction between labour and labour-power—this was Marx's scenario. In order to show this, the distinction between labour and labour-power is important. Thus, Marx's theory of surplus value is based on this distinction, and at least in the exposition of the accepted theory of transformation this distinction is clear.

Is there no possibility that the distinction between labour and labour-power, and hence exploitation may be obscured by the distribution approach?

4. We did not discuss the joint-production case, but as we discussed elsewhere, the quantitative aspect of the transformation of values into production prices encounters difficulty, if the joint-production system is dealt with.¹²⁾ The qualitative aspect such as the fundamental Marxian theorem will hold, nevertheless. This implies that the qualitative aspect is more important to the transformation theory.

We may conclude that the Lipietz-type theory of transformation, as it stands, may not be said to be as an extension of the transformation theory, and hence cannot be said to replace the accepted one.

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12) Dumenil [84] criticised Lipietz from the standpoint of joint-production system, although Dumenil [84] expressed his fundamental support to Lipietz's argument. We do not discuss this point further.

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