Sraffa in the Light of Marx

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I. Introduction

The theory of price is one of the major problems in economics which have long been discussed by many scholars. It is no wonder that from the outset theories of price have had close relationship with the theory of capital and distribution.

Neo-classical economics is one of the major schools which have occupied the stardom. Its marginal productivity theory is exposed to criticism, however, that it would be valid if the capital-labour ratio were to be uniform throughout the economy. In the course of criticism, Sraffa presented the theory of the standard commodity.

Whilst, in the camp of Marxian school, the transformation theory, which had long been brought to a standstill, found a new way, a new direction to go. That is, a mathematical solution to the transformation problem was obtained, and many important propositions were established.

Now, it will be interesting to examine Sraffa's theory of the standard commodity from the angle of Marx's transformation theory. First, one shall be concerned with a brief summary of the theory of the standard commodity in the no joint-production case. Some limits of the theory will be pointed out. Secondly, the location of Sraffa's theory will be made clear from the angle of the transformation of values into prices. Lastly, the main discussion will be confirmed in the case of joint-production. Thus, one will be able to deduce a conclusion upon Sraffa and Marx.

The assumptions explicitly required in the subsequent analysis are as follows.

(♯1) The population of the economy is divided into two major classes—the capitalist class and the working class.

(♯2) Capitalists acquire profit, and workers earn wages. Other forms of income than profit and wages are disregarded.

(#3) The profit-rate and the rate of exploitation are uniform throughout the economy.

(#4) Each commodity is reproduced in a unit period.

(#5) The economy is productive.

(#6) Direct labour input is indispensable in production.

(#7) n processes are employed in the economy, producing m kinds of commodities.

Notations used are as follows.

A	$m\times n$	:	input coefficients matrix
B	$m\times n$	:	output coefficients matrix
L	$1\times n$	:	labour coefficients vector
f	$m\times 1$	:	consumption goods bundle per a unit of labour
w	$1\times m$	:	value vector
p	$1\times m$	:	price vector
r		:	value rate of profit
π		:	profit-rate
R		:	maximum possible profit-rate
$\omega=wf$		:	real wage-rate
$\bar{\omega}=pf$		:	wage-rate
$\tilde{\omega}=\omega/RwAx$		:	standard real wage-rate
$\hat{\omega}=\bar{\omega}/RpAx$		:	standard wage-rate
x	$n\times 1$	:	intensity vector of the standard system
y	$n\times 1$	:	intensity vector of the capacity growth
e		:	rate of exploitation
λ_K		:	Frobenius-root of a matrix K
θ_K		:	right-side Frobenius-vector of a matrix K
θ_K'		:	left-side Frobenius-vector of a matrix K

$M=A+fL$, $D=B-A$, $D_{(R)}=B-(1+R)A$.

Employing the notations above, one can add some mathematical expressions.

In the first place, let us assume that

(#8) $$A, B\geq \mathbf{0}, \quad f\geq 0^m, \quad L\geq 0_n.$$

Productiveness of the economy can be represented as:

(#5′) there exists $z\geq 0^n$ such that $Bz>Az$.

Indispensability of labour can be expressed as:

(#6′) $Lz>o$ for $z\geq 0^n$ such that $Bz>Az$.

Furthermore, let us assume that

(#9) if A is a square matrix, then A is indecomposable and primitive.

This usual assumption plays an important role in sections II and III.

If (#9) holds, the productiveness of the economy can be expressed by:

(#5″) $$\lambda_A<1.$$

This condition will be frequently used in sections II and III. Hence, the explicit reference to it may not be necessarily made in the following.

II. Sraffa's standard commodity—a brief summary and comments

Let us begin with a brief mathematical summary of Sraffa's theory of the standard commodity in the no joint-production case without choice of techniques and fixed capital. Then, $m=n$, and A and B are square matrices of dimension n. Moreover, one can write $B=I$. An intensity vector z can be identified with the output vector. As mentioned before, (#9) will be taken into account.

Capital defined by Sraffa is the conglomeration of the means of production. That is, capital does not contain labour-power. According to this definition, Sraffa's price system is represented by the formula as

$$p=(1+\pi)pA+\bar{\omega}L. \qquad (1)$$

Given $\bar{\omega}$, one has

$$p=\bar{\omega}L(I-(1+\pi)A)^{-1}. \qquad (2)$$

As well known, there exists an equilibrium solution: p and π are determined simultaneously as functions of $\bar{\omega}$, the relationship between the two is illustrated by the wage-profit curve.

Then, the following proposition presents the condition of prices being independent of the profit-rate.

Proposition 1.

$$dp/d\pi=o \Longleftrightarrow p(\lambda I-A)=0_n.$$

Proof.

Since the sufficiency is self-evident, one has only to prove the necessity of the proposition.

Differentiating (1) with respect to π, one has

$$dp/d\pi(I-(1+\pi)A)=pA+(d\bar{\omega}/d\pi)L=0_n.$$

Then, it follows that one has $d^2\bar{\omega}/d\pi^2=o$ from

$$(dp/d\pi)A+(d^2\bar{\omega}/d\pi^2)L=0_n.$$

Therefore, $d\bar{\omega}/d\pi$ is a constant, and

$$\bar{\omega}=(d\bar{\omega}/d\pi)\pi+k,$$

where k is a constant. Now, one has

$$\begin{aligned} p&=(1+\pi)pA+((d\bar{\omega}/d\pi)\pi+k)L \\ &-pA+kL. \end{aligned}$$

Therefore,

$$\begin{aligned} (1+\pi)pA&=p-\bar{\omega}L \\ &=(1-(\bar{\omega}/k))p+(\bar{\omega}/k)pA \end{aligned}$$

and hence,

$$p\propto pA. \qquad \text{Q.E.D.}$$

The condition $p(\lambda I-A)=0_n$ is invariably accompanied by its dual. That is, one has the following two conditions:

$$p(\lambda I-A)=0_n \qquad (3)$$

and

$$(\lambda I-A)z=0^n. \qquad (4)$$

Here, it is easily seen that $\lambda_A = 1/(1+R)$. By dealing with the implications of these two conditions, one can get the following propositions. Since the first condition has something to do with value, it is necessary to define value in advance:

$$w = wA + L = L(I-A)^{-1}. \quad (5)$$

Proposition 2.

The following six conditions are all equivalent:

i) the actual price vector fulfils (3),

ii) the actual price vector is proportional to the value vector,

iii) L is the left-side eigen-vector of A,

iv) the organic compositions of production lines are uniform,

v) the value vector is proportional to L,

vi) the actual price vector is proportional to L.

Proof.

First, one soon gets that iii), v) and vi) are equivalent. In fact, $p \propto L$ implies $L \propto LA$, and $L \propto LA$ implies $p = \bar{\omega} L(I-(1+\pi)A)^{-1} \propto L$. Likewise, $L \propto LA$ implies $w \propto L$ and $w \propto L$ implies $L \propto LA$.

Secondly, if $L \propto LA$, then $wA\hat{L}^{-1} = L(I-A)^{-1}A\hat{L}^{-1} \propto L\hat{L}^{-1}$. So, iv) holds. If one has iv), then $wA \propto L$ and hence, $w \propto L$. Thus, one gets v).

Thirdly, if $L \propto LA$, then one has $p \propto L$ and $w \propto L$, and hence ii), from which it follows that one has i) as shown in the proof of Proposition 1. It is trivial that i) implies vi).

Q. E. D.

Proposition 3.

If any one of the above mentioned six conditions holds, then the relation between π and $\bar{\omega}$ becomes linear for any output vector:

$$\pi = R(1-\bar{\omega}). \quad (6)$$

Nextly, from the dual condition (4), one can define the standard commodity x as the bundle of commodities satisfying

$$x = (1+R)Ax.$$

An economy in which production takes place so as to yield outputs proportional to the standard commodity is called the Sraffian economy. The principal implication of the standard commodity is made clear by the following proposition.

Proposition 4.

Suppose $Lx = 1$. If measured by the standard commodity, then

$$\pi = R(1-\hat{\omega}). \quad (7)$$

for any price system.

Proof.

In fact, it is easily seen that the formula

$$\begin{aligned}\pi &= (px - pAx - \bar{\omega}Lx)/pAx \\ &= R(1-\hat{\omega})\end{aligned}$$

is obtained.

Propositions 3 and 4 may be summarised in the following way: $p(\lambda 1-A)z=o$ implies that $d\bar{\omega}/d\pi=$const. Thus, starting from a necessary and sufficient condition of prices' being independent of distribution, one gets sufficieet conditions for the linearity of the wage-profit curve.

The standard commodity plays, in the first place, a role of weights of aggregation. In order to clarify this, let us consider the case $f=\theta_A$. Suppose that the economy is in the growth equilibrium with no capitalist consumption as:

$$y=(1+g)Ay+fLy.$$

It then follows that the equilibrium output y is proportional to f, and $g=\pi$. The equilibrium situation is precisely similar to the Sraffian economy.

A true dual case of the standard-commodity based analysis is the description of the growth-consumption frontier in terms of prices determined by

$$p^M=(1+\bar{R})p^M A.$$

Suppose that the economy is in the state of balanced growth as

$$y=(1+g)Ay+cfLy.$$

Let $\omega^M=p^M f$, $k=c\omega^M$ and $\hat{k}=k/\bar{R}p^M Ax$, and one has

$$g=\bar{R}(1-\hat{k}).$$

(Needless to say, in the present case, one has $R=\bar{R}$.)

From the above, one can see that before prices are known it is impossible to evaluate the standard national income $RpAx$ and hence one cannot transform $\bar{\omega}$ into $\hat{\omega}$. That is to say, prices and the profit-rate should be determined before the calculation of the standard national income. The standard commodity is not related to the determination of the profit-rate and relative prices. Even in the rare case of actual outputs' being proportional to x, the situation does not change.

Thus, it can be said that Sraffa's theory of the standard commodity does not clarify the mechanism determining the profit-rate and relative prices simulaneously. The theory of the standard commodity may be said, however, to be a descriptive theory of distribution. That is, the standard commodity makes it possible to describe distribution independently of prices, when prices and the profit-rate are already determined. What Sraffa presents is a kind of the ideal numeraire or the aggregator. This is the place of Sraffa's theory of the standard commodity in the theory of price.

Therefore, the determination of relative prices and the profit-rate cannot generally rest on the standard commodity. Moreover, the standard commodity itself should stand on a solid basis of a generalised theory of price which delineates the determination of the profit-rate and relative prices, because whether or not the description of distribution is correct depends on that basis.

In the following section, the theory of the standard commodity will be discussed from the view point of Marx's transformation of values into prices.

III. Marx's transformation theory and the standard commodity

As well known, Marx, taking into account the whole process of reproduction, presented the theory of the transformation of values into prices. The aim of this section is, along the line of Marx's approach, to provide the transformation formula for Sraffa's price system and to make clear the relation between the capacity outputs and the standard commodity. In the following, the value system, Marx's production price system and Sraffa's price system will be discussed.

Value system.

Marx's system of valuation of commodities is twofold. It is the so-called dual system. The first, basic system of valuation is the value system, which is represented, as mentioned before, by:

$$w=wA+L=L(I-A)^{-1}.$$

The value system of commodities is independent of distribution, once choice of techniques is made.

However, commodities are not exchanged by their values. Values are transformed into prices, by which commodities are exchanged.

Marx's production price system.

(1) The case in which A, L, and f are given.

When A, L and f are given, the transformation of values into prices is expressed by the formulae as

$$\begin{aligned} w^{t+1}&=(1+\pi^t)w^tM \\ 1+\pi^t&=w^tz/w^tMz, \qquad w^0=w, \end{aligned} \tag{8}$$

for any outputs $z\geq 0^n$. (Okishio, 1973; 1974).

Production prices are expressed as

$$p=(1+\pi)pM. \tag{9}$$

Since all the elements of M are given, the profit-rate and prices are determined respectively as

$$\pi=(1/\lambda_M)-1$$

and as

$$p=\theta_M'.$$

Here, π can be regarded as a continuos function of f.

Now one has the following proposition.

Proposition 5.

The sequence generated by (8) converges to (9), yielding $p^*z=wz$, where $p^*\propto\theta_M'$. As for a mathematical proof, see Okishio (1973; 1974).

This proposition shows the well-known equality of total price and total value. It should be observed that prices thus established have the absolute magnitudes in the labour-ualue dimension. In this sense, they must not be identified with relative prices in general. In the following arguments, p^* will be used in this sense.

Whereas, the capacity outputs are determined as

$$y=(1+\pi)My.$$

It is easily seen that y depends on f.

If one aggregates by using the capacity output as weights, one gets:

Proposition 6.

(i) $$p^*y=wy, \qquad pZy-p^*My=wy-wMy.$$

(ii) If any two of $p^*y=wy$, $p^*y-p^*My=wy-wMy$ and $p^*My=wMy$ hold, then the remaining equality holds true.

(iii) On the capacity outputs, $r=\pi$.

As for a proof of (i) and (iii), see, e.g. Morishima (1973). (ii) is trivial.

The above (iii) shows that distribution in the world of prices is identical with that in the world of values, if one evaluates distribution on the capacity outputs.

In the next place, let us use the standard commodity as the aggregator.

Proposition 7.

(i) $$p^*Ax=wAx, \qquad Rp^*Ax=Lx.$$

(ii) $$\pi=R(1-\bar{\omega})/(1+\bar{\omega}R),$$
$$r=R(1-\omega)/(1+\omega R).$$

In fact, in view of the definition of x and Proposition 5, (i) is self-evident. And, direct culculation of

$$\pi=(p^*x-p^*Ax-\bar{\omega}Lx)/p^*Mx$$

and

$$r=(wx-wAx-\omega Lx)/wMx$$

leads to the results in (ii).

One may call this proposition a quasi-transformation theorem by the standard commodity in the light of (i). The conclusion (ii) shows that distribution in the worlds of values and prices is parallel. That is, the two curves, $\pi=\pi(\bar{\omega})$ and $r=r(\omega)$, are of the same shape with identical domains and ranges. There is a one-to-one correspondence between π and r, which may be called a quasi Morishima=Seton correspondence.

(2) The case in which A, L and $\bar{\omega}$ are given.

The transformation formulae are expressed by

$$w^{t+1}=(1+\pi^t)(w^tA+\bar{\omega}^tL), \qquad 1+\pi^t=w^tz/(w^tAz+\bar{\omega}^tLz) \tag{10}$$

for any outputs z, where $w^0=w$. Whereas, prices are determined as

$$p=(1+\pi)(pA+\bar{\omega}L)$$

and hence,

$$p=\bar{\omega}L(I/(1+\pi)-A)^{-1}.$$

It should be noted that f in (10) is determined by $\bar{\omega}=pf$. Obviously, f may not be determined uniquely for a given magnitude of $\bar{\omega}$. By taking any f satisfying $\bar{\omega}=pf$ and by fixing it, one can show the convergence of the iteration in a similar fashion as before.

Hence, one may obtain the similar propositions as before. This case is essentially equal to the preceding one.

Sraffa's price system.

Since f plays no important role in the formation of Sraffa's price system, one has only to deal with the case in which A, L and $\bar{\omega}$ are given.

The transformation formulae are represented as

$$w^{t+1}=(1+\pi^t)w^tA+\bar{\omega}L$$
$$1+\pi^t=(w^tz-\bar{\omega}Lz)/w^tAz \tag{11}$$

for some outputs $z\geq 0^n$, where $w^0=w$.

Sraffa's price system is expressed as in section II by

$$p=(1+\pi)pA+\bar{\omega}L.$$

Now, one has the following proposition.

Proposition 8.

(i) $p^{**}x=wx$ is epuivalent to $p^{**}Ax=wAx$ and $RpAx=Lx$, where p^{**} is a price vector satisfying Sraffa's price equation (1).

(ii) $\pi=R(1-\bar{\omega}),\qquad r=R(1-\omega).$

Proof.

Consider the iteration with $z=x$, i.e.

$$w^{t+1}=(w^tx-\bar{\omega}Lx)w^tA/w^tAx+\bar{\omega}L. \tag{12}$$

It is easily seen that

$$w^tx=w^{t-1}x=\cdots=w^0x. \tag{13}$$

Then, postmultiplying (12) by x, one has

$$w^{t+1}x=(1+\pi^t)w^tAx+\bar{\omega}Lx,$$

and hence,

$$\begin{aligned}1+\pi^t&=(w^{t+1}x-\bar{\omega}Lx)/w^tAx\\&=(w^0x-\bar{\omega}Lx)(1+R)/w^0x\\&=\text{const.},\end{aligned}$$

in the light of (13).

Therefore, the iteration formula becomes

$$w^{t+1}=(1+\pi)w^tA+\bar{\omega}L.$$

Repeating this, one gets

$$w^{t+1}=(1+\pi)^{t+1}w^0A^{t+1}+\bar{\omega}L\sum_{k=0}^{t-1}(1+\pi)^kA^k.$$

Since one has $\lambda_A<1$ and $1+\pi<1+R$, it follows

$$p^{**}=\lim_{t\to\infty}w^{t+1}=\bar{\omega}L(I-(1+\pi)A)^{-1}$$

from $\lim_{t\to\infty}A^{t+1}=\mathbf{0}$ and $\sum_{t=0}^{\infty}(1+\pi)^tA^t=(I-(1+\pi)A)^{-1}$

Thus, the workability of the iteration was shown possible. The remaining part of the proof is trivial. Q. E. D.

This proposition is the core of the argument in this section. (i) shows that if one introduces Marxian labour theory of value as the basis of the standard commodity, the standard national income is equal to the amount of labour directly expended. Moreover, the two equations in (ii) show that if one employs p^{**} one can describe distribution, independent of relative prices, parallely in the worlds of values and prices by using the actual magnitudes of ω and $\bar{\omega}$. That is, the two lines, $\pi=\pi(\bar{\omega})$ and $r=r(\omega)$, are identical. There exists a one-to-one correspondence between π and r, which may be termed a specialised quasi Morishima=Seton correspondence.

This must be said a remarkable feature of the standard commodity as the aggregator. If one takes into account the value system behind the price system, one can reduce Sraffa's (6) to (5) for any set of techniques. Thus, the standard commodity can be regarded as an important aggregator.

Finally, one can establish the fundamental Marxian theorem.

Proposition 9. $\pi>o$ if and only if $e(x)>o$, where $e(x)$ denotes the dependence of e on x.

In fact, from the above proposition, one has $\pi>o$ if and only if $r>o$. Since one has

$$r=R\frac{e}{1+e}$$

in view of $(1+e)\omega=1$, one gets the conclusion.

As for the relationship between the capacity outputs and the standard commodity, one can say as follows.

As made clear in the case of Marx's production price system, the capacity outputs are not independent of f or $\bar{\omega}$. If one considers this on the wage-profit curve, f and the capcity outputs change in general, as $\bar{\omega}$ changes. Although f and y are not continuous functions of $\bar{\omega}$, f and y converge respectively to 0^n and x, as $\bar{\omega}$ converges to o. That is to say, the standard commodity is a special case of the capacity outputs that f is equal to 0^n.

Consider a wage-profit curve in the $(\bar{\omega},\pi)$ space. The standard commodity corresponds to the extreme point (o,R) on the wage-profit curve. The other extreme point corresponds to the case that prices are proportional to values. Unambiguously, these two points are the dual extreme points of the capacity outputs. Therefore, it can be concluded that the theory of the standard commodity is a specialised theory of the transformation on the capacity outputs.

IV. The joint-production case

Sraffa's view on the mechanism through which relative prices and the profit-rate are determined simultaneously is somehow similar to von Neumann's theory from the angle of the mathematical framework. Nevertheless, Sraffa's theory is conceptually different from von Neumann's in the sense that Sraffa dealt with the equation system.

If one considers the determination of the relative price system in the joint-production case, however, it appears that Sraffa's theory will come up against difficulties.

Sraffa starts from the usual equilibrium in which the uniform rate of profit and the uniform wage-rate are prevailing, and he presents the standard commodity which enables us to describe distribution independently of relative prices. Since the productiveness of the economy without joint-production and alternative techniques ensures the existence of the non negative standard cemmodity, this description can be done as seen before. Once joint-production is permitted, however, the standard commodity is not necessarily guaranteed to exist. In other words, the existence of the relative price system with a positive profit-rate and the uniform wage-rate does not necessarily imply the existence of the standard commodity. Then, Sraffa's theory of the standard commodity will fail, unless one has some additional assumptions.

The purpose of this section is to make clear conditions under which the standard commodity becomes viable in the joint-production case.

In a usual joint-production system with m kinds of commodities and n processes, the productiveness of the economy can be expressed as: there exists $z \geq 0^n$ such that $Bz > Az$.

The price system can be represented as:

$$pB = (1+\pi)pA + \bar{\omega}L \tag{14}$$

with $p > 0_m$ and $\pi > o$ for $\bar{\omega} \geqq o$.

Suppose that the following three conditions on the technology set are fulfilled.

(♯10) There exist $p > 0_m$ and $R > o$ such that

$$pB = (1+R)pA.$$

(♯11) $$\operatorname{rank} D = \operatorname{rank}\binom{D}{-L}.$$

(♯12) $$\operatorname{rank} D_{(R)} < n.$$

The first condition may be called the maximum profitability condition, *MPfC*. This means that if there is no wages, positive profit is possible. This may be regarded as an economically plausible condition.

The second one ensures the existence of value. That is, (♯11) is equivalent to that there exists w such that

$$wB = wA + L.$$

The third condition guarantees the existence of the standard commodity. In other words, this implies that there exists $x \neq 0$ satisfying

$$Bx = (1+R)Ax.$$

Note that, however, $x \geq 0$ is not ensured by (♯12).

With these additional conditions satisfied, one can extend the similar propositions as in the preceding sections to those in the case of joint-production.

Repeating the central thesis, one can say that $p^{**}Bx = wBx$ is equivalent to $Rp^{**}Ax = Lx$ and $p^{**}Ax = wAx$, and hence $\pi = R(1-\bar{\omega})$ and $r = R(1-\omega)$ hold. The other conclusions established in the preceding sections are also valid in the joint-production case.

The conclusion that Sraffa's theory of the standard commodity is a specilised theory of transformation will be reinforced by the following remark.

In the no joint-production case without alternative techniques, $\bar{\omega}$ can be transformed into $\hat{\omega}$ because $RpAx$ $(=Lx) > o$ from $x \geq 0^n$ and assumption (5). As well known, if $z > Az$ for $z \geq 0^n$, then $z > Az$ for any $z \geq 0^n$. Hence, $x \geq 0^n$ is automatically guaranteed from the productiveness of the economy in this case.

In the joint-production case, however, the standard commodity is no longer non-negative, and hence $RpAx\ (=Lx)>o$ is not ensured unconditionally, even if x exists. Hence, $\bar{\omega}$ cannot be transformed into $\hat{\omega}$ in the case of $RpAx<o$. If one takes into account the equality of total value and total price, this difficulty can be overcome.

In order to avoid the introduction of the assumptions (#10)-(#12), one may permit subsystems defined by inequalities á la von Neumann to supplement Sraffa's analysis.

Let us consider two sub-systems

$$Bx\geq(1+R)Ax$$

and

$$p^M B\leq(1+\bar{R})p^M A,$$

where R is maximised and $\bar{R}$ is minimised. It is not stringent to consider that these two inequalities yield $x\geq 0^n$, $p^M\geq 0_m$, $R>o$ and $\bar{R}>o$. If this Sraffa=von Neumann approach is admitted, some of the results established in a normal no joint-production case discussed so far can be resurrected. That is, the wage-profit curve and the growth-consumption frontier can be reestablished in a similar way as

$$\pi\geq R(1-\hat{\omega})$$

and

$$g\leq\bar{R}(1-\hat{k}),$$

where g is maximised in

$$By\geq(1+g)Ay+fLy.$$

It is not difficult to show that one has a similar quasi Morishima=Seton correspondence and the fundamental Marxian theorem.

Take f such that $pf=\bar{\omega}$. Let values be defined á la Morishima by

$$\begin{aligned}&\text{maximise} && wfLx\\ &\text{subject to} && wB\leq wA+L,\\ &&& w\geq 0_m,\end{aligned}$$

the dual problem of which is

$$\begin{aligned}&\text{minimise} && Lz\\ &\text{subject to} && Bz\geq Az+fLx,\\ &&& z\geq 0^n.\end{aligned}$$

Let w^0 and z^0 be optimum solutions, and let us presuppose $Lx=1$. Since one has $Lz^0=w^0f$, the rate of exploitation can be expressed by

$$e(x)=\frac{1}{Lz^0}-1=\frac{1}{w^0f}-1.$$

Now, one can establish:

Proposition 10. Suppose that $Lx=1$. Then, there is a one-to-one correspondence between $\pi_{\min}$ and $r_{\min}$:

$$\pi_{\min}=R(1-\hat{\omega}),$$

and

$$r_{\min}=R(1-\bar{\omega}).$$

Proposition 11. $\pi>o$ if and only if $e(x)>o$.

Thus, starting from the system of price equations (14), one can formulate Sraffa's theory of distribution without violating the spirit of Sraffa.

V. Concluding remarks

Assessing Sraffa's theory of the standard commodity, one may say as follows. The standard commodity has nothing to do with the determination of relative prices and the profit-rate. The standard commodity makes it possible, however, to express distribution independently of relative prices, so that the theory of the standard commodity can be regarded as a descriptive theory of distribution. Moreover, if the concept of value is accepted hehind the standard commodity, Sraffa's theory can survive as a specialised theory of transformation.

Sraffa rather follows Ricardo, who defined political economy as a science to grasp the laws of distribution. While, Marx's economics aims at clarifying the laws of motion of the capitalist economy, and thus production, exchange, distribution and consumption, or, in one word, the whole process of reproduction of the capitalist economy.

Therefore, it constitutes one of the most vital part of Marx's economics to delineate not only the establishment of the uniform profit-rate but also the source of profit, while in Sraffa's theory, the problems are argued primarily from the angle of distribution rather than from that of reproduction. It is evident that Marx went further than Ricardo or Sraffa.

In summary, Marx's theory of transformation is a generalised theory of production and distribution, upon which Sraffa's theory can rest. It is no wonder that in the course of criticism of orthodox economics, the revival of discussions on Marx, called Marx Renaissance, has appeared.

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