Formation of the Leontief Core and the Concept of Pseudo Value

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Introducton—Problem

1. It can be said that the von Neumann economic model represents one of the most generalised frameworks of production theory. The von Neumann economic theory explains the qualitative aspect of equilibrium rather than the quantitative one.

It has been argued in other places that various aspects of Marx's value concept coincide with each other ina Leontief economy: value of a product as the amount of labour embodied or crystallised is equivalent to its valuation, the amount of labour required, which maximises the productivity of labour. In a von Neumann economy, however, these two concepts turn out to be different from each other, and optimum value, which maximises the productivity of labour in a von Neumann economy, is relevant for the explanation of the fundamental Marxian theorem. Nevertheless, as opposed to the value as the amount of labour embodied or crystallised, the optimum value is not so operational, or well-behaved: the optimum value is not unique, and is often zero.[1] Hence, it is interesting to find a more operational value concept.

A generalisation of the first definition of value does not sound easy in a von Neumann economy, in so far as the orthodox Marxian value equation does not fulfill its solvability condition. Is there really no way to generalise the concept of value as a solution of the value equation?

The purpose of this short paper is to present another attempt to evaluate an operational type of value so as to explain the fundamental Marxian theorem.

2. In order to make clear the future procedure developed in this paper, let us illustrate the point of discussion by using a simple numerical example.

Consider the input-output relationships expressed by the following output matrix B, input matrix A and labour vector L.

$$B=\begin{bmatrix}2 & 4\\ 3 & 2\end{bmatrix},\quad A=\begin{bmatrix}1/3 & 2\\ 2 & 2/3\end{bmatrix},\quad L=(1,1).$$

Suppose that the wage goods bundle is represented by

$$F=\begin{bmatrix}1\\ 0\end{bmatrix}.$$

Then, the production prices (p_1, p_2) and the profit rate π are determined by:

$$(p_1,p_2)\begin{bmatrix}2 & 4\\ 3 & 2\end{bmatrix}=(p_1,p_2)\left[\begin{bmatrix}1/3 & 2\\ 2 & 2/3\end{bmatrix}+\begin{bmatrix}1 & 1\\ 0 & 0\end{bmatrix}\right](1+\pi).$$

By putting $p_2=1$, we get $p_1=2$ and $\pi=1/2$.

Let us disaggregate the two processes. Each of the two is a joint-production process, but can be regarded as a synthesis of processes which produce only single type of goods. Suppose that one unit of each process is operated. Under the ongoing prices and the profit rate, the total production of the first process is evaluated as

$$2\times 2+3\times 1=7.$$

Then, we may regard that 4/7 of the inputs of the first process are used for the production of good 1, while 3/7 for good 2. Thus, from the first process we can form the following subeconomy 1:

$$A_1{}^*=\left[4/7\begin{bmatrix}1/3\\ 2\end{bmatrix},3/7\begin{bmatrix}1/3\\ 2\end{bmatrix}\right],\quad L_1{}^*=(4/7,3/7),\quad B_1{}^*=\begin{bmatrix}2 & 0\\ 0 & 3\end{bmatrix}.$$

Likewise, from the second process, we get subeconomy 2:

$$A_2{}^*=\left[4/5\begin{bmatrix}2\\ 2/3\end{bmatrix},1/5\begin{bmatrix}2\\ 2/3\end{bmatrix}\right],\quad L_2{}^*=(4/5,1/),\quad B_2{}^*=\begin{bmatrix}4 & 0\\ 0 & 2\end{bmatrix}.$$

Now, let us aggregate these two subeconomies: we have

$$A_1{}^*=A_1{}^*+A_2{}^*=\frac{1}{105}\begin{bmatrix}188 & 57\\ 86 & 104\end{bmatrix},\quad L^*=L_1{}^*+L_2{}^*=(48/35,\ 22/35),$$

$$B^*=B_1{}^*+B_2{}^*=\begin{bmatrix}6 & 0\\ 0 & 5\end{bmatrix}.$$

It is not difficult to see that the above transformation is always possible and that once prices and the profit rate are fixed, the transformed input-output relationships are uniquely determined.

Moreover, using the above B^*, A^* and L^*, we can evaluate prices and the profit rate of the original von Neumann economy. That is to say, from the system of equations as

$$(p_1,p_2)\begin{bmatrix}6 & 0\\ 0 & 5\end{bmatrix}=(1+\pi)(p_1,p_2)\left[\frac{1}{105}\begin{bmatrix}188 & 57\\ 86 & 104\end{bmatrix}+\begin{bmatrix}48/35 & 22/35\\ 0 & 0\end{bmatrix}\right],$$

and

$$p_2=1,$$

we get $p_1=2$ and $\pi=1/2$. This fact implies that the above transformation preserves prices and the profit rate.

In what follows, the above transformation from (B, A, L) to (B^*, A^*, L^*) will be called the core transformation, and the transformed input-output relationship may be said to constitute the Leontief core of the given economy.

Since the core transformation does not change the profit rate and the prices of the original von Neumann economy, we may evaluate the values of goods from B^*, A^* and L^*: we can apply the results established with respect to Marx's theory of value in the Leontief economy case. Since the Leontief core depend on distribution, the values evaluated in the Leontief core are also dependent on distribution. Hence, we shall call values evaluated from B^*, A^* and L^* pseudo-values. It is interesting to ask whether Marx's theroy of value will be resurrected in terms of pseudo-value.

In § 1, we shall develop formally the core transformation, and in § 2 we shall discuss the quantity system and growth in relation to the core transformation. Thus, we shall make clear the properties of the core transformation. § 3 will be devoted to Marx's theory of value in terms of pseudo-value and its implications, both theoretical and practical.

The following mathematical notations are adopted:

Z^j : j-th row of matrix Z

$Y_i,(Y^j)_i$: i-th element of vector Y (Y^j)

$\hat{z}$: diagonal matrix composed of elements of vector z.

$1^m,(1_m)$: m-clolumn (row) vector, components of which are all unity. (summation vector).

§ 1. The price system and the Leontief core—the core transformation

1. Consider an ordinary joint-production system with m types of good and n processes. It is presuposed that the input-output relationships of the system are of point-input and point-output type. The heterogeneous labour case will be excluded hereafter.

Notations employed are as follows:

A $m\times n$: input matrix

B $m\times n$: output matrix

L $1\times n$: labour vector

F $m\times 1$: wage goods bundle (per unit of labour).

p $1\times m$: price vector

π : profit rate

α_{ij} : core weight of goods j in process i.

Now, start from the equilibrium price system described by:

(1) $$pB=(1+\pi)p(A+FL).$$

Using the solutions of (1), p and π, we can define the core weight. Take process i, i.e, B^i, A^i and L_i. The core weight is represented by

(2) $$\alpha_{ij}=p_jB_j{}^j/pB^j,$$

so that the inputs that amount to $\alpha_{ij}A_i{}^i$ and $\alpha_{ij}L_i$ are regarded as the inputs required for the production of $B_j{}^i$ units of goods j. Thus, we can transform process i into subeconomy i which consists of no joint-production proceses. Here, let us write:

B_i^* $m\times m$: output matrix of subeconomy i

A_i^* $m \times m$: input matrix of subeconomy i

L_i^* $1 \times m$: labour vector of subeconomy i.

These are represented by

(3)
$$B_i^* = (B_1{}^i e^1, \cdots, B_m{}^i e^m) = \hat{B}^i,$$
$$A_i^* = (\alpha_{i1} A^i, \cdots, \alpha_{im} A^i),$$
$$L_i^* = (\alpha_{i1} L_i, \cdots, \alpha_{im} L_i).$$

From all the processes in the economy, we have the set of subeconomies 1 through n. Aggregate B_i^*s, A_i^*s and L_i^*s respectively, and we can define:

$B^* = \sum B_i^*$: core output matrix

$A^* = \sum A_i^*$: core input matrix

$L^* = \sum L_i^*$: core labour vector.[2),3)]

The transformation $(B, A, L) \rightarrow (B^*, A^*, L^*)$ is called the core transformation. Moreover,

Definition 1. (Leontief core) The no joint-production system described by (B^*, A^*, L^*) is called the Leontief core of the economy.

Prices and the profit rate in the Leontief core are also termed core prices and the core profit rate respectively. Let us denote:

p^* $1 \times m$: core price vector

π^* : core profit rate.

These are determined by the subsequent equation:

(4)
$$p^* B^* = (1+\pi^*) p^* (A^* + FL^*).$$

What is the relationship between prices and the profit rate of the original von Neumann economy and those of its Leontief core?

Proposition 1. p and π satisfying (1) also satisfy (4):

$$\{(p, \pi) |\ pB = (1+\pi) p (A+FL)\} \subset \{(p^*, \pi^*) |\ p^* B^* = (1+\pi^*) p^* (A^* + FL^*)\}$$

Proof.

This is a matter of definition. Algebraic manipulation enables us to confirm this. In fact, rewrite (1) according to the above transformation, and equation $pB^i = (1+\pi) p (A^i + FL_i)$ is transformed into

(5)
$$p\hat{B}^i = (1+\pi) p (A_i^* + FL_i^*).$$

Namely,

(6)
$$pB_i^* = (1+\pi) p \{(A^i, \cdots, A^i)\hat{\alpha}^i + F(L_i, \cdots, L_i)\hat{\alpha}^i\},$$

where, $\alpha^i = (\alpha_{i1}, \cdots, \alpha_{im})$.

The system of subeconomies is represented by

(7)
$$p(B_1^*, \cdots, B_n^*) = (1+\pi) p \{(\bar{A}^1 \hat{\alpha}^1, \cdots, \bar{A}^n \hat{\alpha}^n) + F(\bar{L}_1 \hat{\alpha}^1, \cdots, \bar{L}_n \hat{\alpha}^n)\},$$

where $\bar{A} = (A^i, \cdots, A^i)$, $\bar{L}_i = (L_i, \cdots, L_i)$.

By aggregating this, we get

(8)
$$p(B_1^*, \cdots, B_n^*) \cdot \bar{I}_n{}^m = (1+\pi) p \{\bar{A}^1, \cdots, \bar{A}^n)\hat{\alpha} + F(\bar{L}_1, \cdots, \bar{L}_n)\hat{\alpha}\} \cdot \bar{I}_n{}^m,$$

where $\bar{I}_n{}^m = {}^t(I_m, \cdots, I_m)$, $\alpha = (\alpha^1, \cdots, \alpha^n)$. That is, we have

$$pB^* = (1+\pi) p (A^* + FL^*).$$

Q. E. D.

Therefore, we can invariably transform a von Neumann economy into the Leontief economy associated with it, and thus this transformation yields the unique Leontief core having the equal prices and the profit rate.

2. The above procedure can be extended to include the unequal profit rate case.

Given an arbitrary price vector $\tilde{p}$ satisfying

(9) $$\tilde{p}B=\tilde{p}(I+\hat{\Pi})(A+FL),$$

where,

π_i : productwise profit rate

$\Pi=(\pi_1, \cdots, \pi_m)$.

Let us carry out the core transformation by using $\tilde{p}$ in the evaluation of weights α_{ij}s. Then, we can form, as before, the Leontief core $(B^*(\tilde{p}), A^*(\tilde{p}), L^*(\tilde{p}))$. Here, let us write:

$\tilde{p}^*$ $1\times m$: core price vector

π_i^* : productwise core profit rate

$\Pi^*=(\pi_1^*, \cdots, \pi_m^*)$.

These are determined by

(10) $$\tilde{p}^*B^*(\tilde{p})=\tilde{p}^*(I+\hat{\Pi}^*)\{A^*(\tilde{p})+FL^*(\tilde{p})\}.$$

Then, we can establish:

Proposition 2. $\tilde{p}$ and Π satisfying (9) also fulfill (10):

$$\{(\tilde{p}, \Pi)|\tilde{p}B=\tilde{p}(I+\hat{\Pi})(A+FL)\}\subset\{(\tilde{p}^*, \hat{\Pi}^*)|\tilde{p}^*B^*=\tilde{p}^*(I+\hat{\Pi}^*)(A^*+FL^*)\}.$$

(In fact, the same procedure as in the proof of Proposition 1 yields the conclusion of this proposition. It is not difficult to see that the proof of Proposition 1 does not depend on the mark up factor, $1+\pi$.)

Namely, the core transformation preserves prices and profit rates even if unequal commoditywise profit rates prevail in the economy.

§ 2. The quantity system, growth and the core transformation

1. Let us investigate the quantity side of the economy and further properties of the core transformation.

Let us employ:

x $n\times 1$: intensity vector

s $m\times 1$: net product vector

s $m\times 1$: surplus product vector.

x^* $m\times 1$: intensity vector of the Leontief core

y^* $m\times 1$: net product vector in the Leontief core

s^* $m\times 1$: surplus product vector in the Leontief core.

Now, without loss of generality, we can assume

(A1) $$x=1^n, \qquad Bx=1^m, \qquad x^*=1^m.$$

In other words, each row of the input and output matrices represents the total amounts of input and output of the process concerned, and only one unit of each process is operated in the economy. Moreover, the total output of each type of goods is normalized as unity.[4)]

In the light of the definition of B^*, it soon follows

(11) $$Bx = B^*x^* = 1^m.$$

An economy is said to be productive, if the productiveness condition, $(Pd.\ C)$ that there exists $x \geqq 0^n$ such that $Bx \geqq Ax$, is satisfied. We say, for the sake of preciseness and brevity, that (B, A) is productive, if $(Pd.\ C)$ with respect to B and A is fulfilled.

The net product vectors, y and y^*, are defined by

(12) $$y = Bx - Ax,$$
$$y^* = B^*x^* - A^*x^*,$$

respectively. Then, we can prove:

Proposition 3. $$y^* = y.$$

Proof.

Rewriting y^* in view of the definitions of B^* and A^*, and in the light of the transformation procedure shown by (5) through (8), we get

$$\begin{aligned} y^* &= 1^m - (\bar{A}^1, \cdots, \bar{A}^n)\alpha \bar{I}_n{}^m \cdot 1^m \\ &= 1^m - (\bar{A}^1, \cdots, \bar{A}^n)\alpha 1^{mn} \\ &= 1^m - (\bar{A}^1, \cdots, \bar{A}^n) \cdot {}^t(\alpha^1, \cdots, \alpha^n) \\ &= B1^n - (A^1 + \cdots + A^n) \\ &= Bx - Ax = y. \end{aligned}$$

Q. E. D.

As an immediate corollary of the above, we have:

Proposition 4. That (B, A) is productive implies that (B^*, A^*) is productive.

(The proof is trivial.)

We can deduce the similar conclusion on the amount of labour employed and the surplus product. The surplus product vector can be defined by

(13) $$s = Bx - (A + FL)x,$$
$$s^* = B^*x^* - (A^* + FL^*)x^*,$$

respectively in a von Neumann economy and in its Leontief core. Now, we obtain:

Proposition 5. (i) $$L^*x^* = Lx,$$

(ii) $$s^* = s.$$

Proof.

(i) The proof is corried out by algebraic manipulation. In fact,

$$\begin{aligned} L^*x^* &= (\bar{L}_1\alpha^1, \cdots, \bar{L}_n\alpha^n)\bar{I}_n{}^m \cdot x^* \\ &= (\bar{L}_1, \cdots, \bar{L}_n)\alpha \cdot \bar{I}_n{}^m 1^m \\ &= (\bar{L}_1, \cdots, \bar{L}_n) \cdot {}^t\alpha \\ &= L1^n = Lx. \end{aligned}$$

(ii) This is trivial in the light of Proposition 3 and the above (i). Q. E. D.

2. Let us next consider growth in the von Neumann economy and in its Leontief core. The following notations are employed:

g : equilibrium growth rate

g^* : core equilibrium growth rate

g_i : productwise growth rate

$G=(g_1, \cdots, g_m)$

g_i^* : productwise core growth rate

$G^*=(g_1^*, \cdots, g_m^*)$

U $m\times 1$: capitalist consumption

Suppose first that the economy is in the state of equilibrium growth described by

(14) $$Bx=(1+g)(A+FL)x+U.$$

The corresponding equilibrium growth in the Leontief core is represented by

(15) $$B^*x^*=(1+g^*)(A^*+FL^*)x^*+U.$$

Note that the capitalist consumption in both systems is equal. Now, we can show:

Proposition 6. $g^*=g.$

Proof.

From Proposition 5 (ii), we have

(16) $$(A^*+FL^*)x^*=(A+FL)x.$$

Whilst, by comparing (14) and (15), we get

$$g^*(A^*+FL^*)x^*+U=g(A+FL)x+U$$

also from Proposition 5 (ii). Hence, it follows $g^*=g$. Q. E. D.

Suppose in the next place that the production of each type of good expands at unequal growth rates. In the von Neumann economy we have

(17) $$Bx=(I+\hat{G})(A+FL)x+U,$$

whilst, the counterpart in its Leontief core is represented by

(18) $$B^*x^*=(I+\hat{G}^*)(A+FL^*)x^*+U.$$

The above proposition can be extended to:

Proposition 7. $G^*=G.$

Proof.

In fact, from Proposition 5, we get

$$\hat{G}^*(A^*+FL^*)x^*+U=\hat{G}(A+FL)x+U.$$

In view of (16), we obtain the conclusion. Q. E. D.

3. In summary, the Leontief core is equivalent to its original von Neumann economy in the sense that the productiveness of the system is preserved, and that the net products, the amount of labour employed, the surplus products, the prices of goods, the profit rates and the growth rates are kept unchanged.

The gist of the core transformation lies in disaggragating a joint-production process into no joint-production processes with the equal structure of inputs and hence the equal organic composition of capital.

A Leontief economy is the Leontief core of itself. A generalised Leontief economy, defined as the Leontief economy with alternative no joint-production processes, is also the Leontief core of itself, provided that processes producing the same type of good are aggregated.

Furthermore, it must be observed that in so far as the quantity side of the economy is concerned, the core transformation does not depend on the valuations of goods employed for the determination of weights α_{ij}s. That is. we can define the core transformation for any v ($\geqq 0_m$) without changing the features of the quantity side of the economy.

Given an arbitrary v ($\geqq 0_m$) satisfying

(19) $$vB = v(I+\hat{\Pi})(A+FL).$$

This defines the set:

$$S = \{v|(19)\}.$$

Carry out the core transformation in terms of v, which may be called a generalised core transformation. We then have $B^*(v)$, $A^*(v)$ and $L^*(v)$, which determine the equilibrium prices and the profit rate in the Leontief core:

(20) $$v^*B^*(v) = v^*(I+\pi^*I)\{A^*(v)+FL^*(v)\}.$$

Define the set:

$$S^* = \{v^*|(20)\}.$$

Consider the mapping:

$$\phi\colon S \to S^*.$$

Here, it is not difficult to see that the equilibrium prices and the profit rate of the original von Neumann economy are a fixed point of the above mapping ϕ, because both S and S^* can be circumscribed to a relevant subset of $\boldsymbol{R}_+^m$.

§ 3. Pseudo-value and its implications—Concluding remarks

1. In the light of the discussion so far, it can be said that the Leontief core represents the original von Neumann economy. Various problems can be described in terms of B^*, A^* and L^*. In this section, we shall be concerned with Marx's value concept.

Introduce the following concept:

Definition 2. (Pseudo-value) Value of goods evaluated in the Leontief is called pseudo value.

Let us promise

w^+ $1\times m$: pseudo-value vector.[5]

This is determined by

(20) $$w^+B^* = w^+A^* + L^*.$$

Make the subsequent assumption:

(A2) $$A \geqq 0, \quad B \geqq 0, \quad L \geqq 0_n.$$

Now, we can establish the following theorems.

Theorem I. If (B, A) is productive, then there exists nonnegative pseudo-value, $w^+ \geqq 0_m$.

Proof.

In view of Proposition 4, (B^*, A^*) becomes also productive. Whilst, from (A2), it follows that B^*, A^* and L^* are all nonnegative. Hence, the equation (20) yields nonnegative solutions.

O. E. D.

Let the core rate of surplus value be defined by

(21) $$\mu^+ = \frac{1}{w^+F} - 1$$

in terms of pseudo-value. Then, one can prove:

Theorem II. (Quasi fundamental Marxian theorem) The positivity of the profit rate implies the positivity of the core rate of surplus value: $\pi>0 \Rightarrow \mu^+>0$.

Proof.

By applying the ordinary fundamental Marxian theorem to the Leontief core, we have: $\pi^*>0 \Rightarrow \mu^+>0$. Whilst, in the light of Proposition 1, we have $\pi^*=\pi$. The conclusion soon follows. Q. E. D.

The above two theorems demonstrate that even if Marx's value equation is not solvable in the original von Neumann economy, there always exists a pseudo-value, and that the fundamental Marxian theorem can be established in terms of pseudo-value.

2. In addition to the above major theorems, propositions which obtain in a Leontief economy are also valid in the Leontief core, i.e., in terms of pseudo-value.

It must be observed, however, that the concept of pseudo-value is a price-ridden concept. Propositions obtained in terms of pseudo-value should be read from the price to value dimensions, because pseudo-values are evaluated ex post, but can explain the positivity of the profit rate, because the Leontiet core is equivalent to the original von Neumann economy.

Moreover, it must be emphasized that what is explained is of quality, and not of quantity. It is easy to see that the Morishima-Seton equality holds in the Leontief core. In Leontief economy, the Morishima-Seton equality represents the determination of the profit rate, and hence the regulation of the profit rate by the value dimension, because the equality can be established ex ante. Whilst, in the Leontief core, it is evaluated ex post: it will be tautological to say that the level of the profit rate is determined by the surplus value divided by total capital in terms of pseudo-value.

If pseudo-value is compared with the traditional value, defined in the original von Neumann economy, it is found that pseudo-value is different from value. Suppose that Marx's value equation, $wB=wA+L$, has a positive solution $w>0_m$. Even in such a case, pesudo-value does not coincide with value: $w^+\neq w$ in general.

Returning to the numerical example given in Introduction, we can easily confim this. Constructing the Leontief core of the given system, we have the non-negative pseudo values and the positive core rate of surplus value: in fact, from

$$(w_1^+, w_2^+)\begin{bmatrix}6 & 0\\0 & 5\end{bmatrix}=(w_1^+, w_2^+)\frac{1}{105}\begin{bmatrix}188 & 57\\86 & 104\end{bmatrix}+(48/35, 22/35),$$

we have $(w_1^+, w_2^+)=(0.3659\cdots, 0.2063\cdots)$, and $w_1^+F=w_1^+<1$, which implies $\mu^+>0$.

Whilst, the traditional value can be evaluated as

$$\begin{aligned}(w_1, w_2)&=(1, 1)\begin{bmatrix}5/3 & 2\\1 & 4/3\end{bmatrix}^{-1}\\&=(3/2, -3/2),\end{aligned}$$

and the rate of surplus value is determined by $\mu=\dfrac{1}{wF}-1<0$.

The discrepancy between the traditional value and pseudo-value may be based on the manner in which processes participate in or are related to the determination of value or pseudo-value. Namely, in the traditional Marxian value equation, all the processes participate, on the equal footing, in the determination of value, irrespective of that they contribute in different degrees to the production of various types of good. Whilst, in the case of the pseudo-value equation, the difference in degrees of the contribution of processes to the production of various types of good is taken into account.

It may be said that the core transformation is an inverse transformation from price to value, and that pseudo-value is a limit point of this inverse transformation.

Furthermore, it should be pointed out that the core transformation can be applied to the heterogenous labour case.

3. The above discussion may present a theoretical basis for the understanding of the actual inter-industry table. Even if we start "conceptually" from von Neumann economy, the complete knowledge of input and output matrices will not be available, and hence even the most detailed inter-industry table will be based on aggregation: its entries are evaluated in terms of price. Taking into account the actual procedure of accounting akin to the core transformation, we may say that the inter-industry tables actually compiled are, to a greater extent, the Leontief core of the economy, and that the inter-industry-table-based empirical analysis of value is concerned with the pseudo-value.[6]

Footnotes

1) As for Marx's two definitions of value and the optimum value advocated by Morishima, refer to, e.g., Fujimori Chapters. II–V.

2) Since $\alpha_{ij}s$ are dependent on prices, A^* and L^* also depend on prices. Hence, where necessary, they are expressed as $A^*(p)$ and $L^*(p)$. Whilst, B^* does not depend on prices. Nevertheless, we shall also write $B^*(p)$.

3) Notations with * are related to the Leontief core.

4) Thus, we exclude processes which are not operated and goods which are not produced.

5) In addition to note 3) above, superscript + indicates the value dimension in the Leontief core.

6) In the actual evaluation of cost of joint-products, the core transformation is adopted in accounting, whilst the traditional annuity method is employed so as to evaluate aged fixed capital.

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