Harrodian Equilibrium and Technical Changes

by Yoriaki Fujimori

Introduction

1. Most microscopic analyses of production and distribution so far made do not seem to give a self-contained explanation in the sense that at least one of the important variables should be treated as a given parameter.[1)]

In order to close the system of equations or inequalties to discuss production and distribution, various attempts have been made: one has only to introduce one more independent relation.

Pasinetti proposed that the Cambridge equation should be introduced to close the price system.[2)]

Namely, as Harrod pointed out, there are two types of growth rate: the natural growth rate and the warranted growth rate. In Pasinetti, the natural growth rate is supposed to be given, to which the warranted growth rate is considered to be equalised. With reference to the general equilibrium analysis of Marx's economics, Morishima=Catephores argued this point.

That is, they formulated the natural growth rate as a function of the real wage rate, and defined the Harrodian equilibrium in which the natural growth rate is equalised to the warranted growth rate.

The objective of this short paper is to make some additional points on an effect of technical changes on the Harrodian equilibrium. After the basic feature of the Harrodian equilibrium is reconfirmed, two fundamental types of shift of the Harrodian equilibrium

1) Take Sraffa's system, for instance. Either a profit rate or a real wage rate should be given. The same is true with von Neumann's inequality approach.
2) This point was also mentioned in Fujimori, Ch. VII.

will be illustrated, and their implications will be considered.

2. The following investigation will be made by using the framework of an ordinary Leontief economy: no fixed capital, no alternative technique, no joint-production and no heterogeneous labour: n types of commodity are produced by point-input, point-output techniques. Production of commodities is completed in one unit period.[3] In what follows, those who obtain profit are called capitalists, while those who are paid wages are called workers.

Basic notations are listed below:

A	$n\times n$	: input matrix,
L	$1\times n$	: labour vector,
F	$n\times 1$	: wage goods bundle,
x	$n\times 1$	: output vector,
p	$1\times n$	: price vector,
g		: growth rate,
$\omega=pF$		: wage rate,
α		: capital-saving ratio,
β		: labour-saving ratio,
δ		: growth rate of labour productivity,
$\rho[Z]$		: Frobenius root of matrix Z,
$M=A+FL$		: augmented input matrix.

§ 1. Natural growth, warranted growth and the Harrodian equilibrium.

1. Let us begin with the functional relationship between the natural growth rate and the real wage rate according to Morishima=Catephores.

At a given level of the real wage rate, the workers may be divided into two groups: one is willing to accept that real wage rate, and the other not. Let the ratio of willing workers to the total population be called the *ratio of labour supply,* which can be regarded as a function of the real wage rate.

Assume that those who are willing to work at a given real wage rate are all employed. Then, the above two groups are reduced respectively to the employed and the unemployed.

Suppose that the number of the employed can expand at a given rate, and that of the unemployed decreases at a given rate for some of the unemployed cannot avoid dying.

Let

$f(\omega)$: ratio of labour supply,

$\bar{g}$: growth rate of the employed,

$\bar{\bar{g}}$: growth rate of the unemployed,

g_p : growth rate of population,

g_n : natural growth rate,

3) As for details, see eg. Fujimori, p. 11.

and one can write:

(1) $$g_p = \bar{g}f(\omega) + [1 - f(\omega)]\bar{\bar{g}}.$$

This gives the natural growth rate, if there is no change in the productivity of labour, for a given real wage rate ω:

(2) $$g_n(\omega) = g_p(\omega).$$

Assume

(3) $$\bar{g} > 0 > \bar{\bar{g}},$$

(4) $$f(\omega) \in C^1;\ \frac{df(\omega)}{d\omega} \geqq 0;\ \{f(\omega) | \omega \geqq 0\} = \{0, 1\},$$

and it follows that

(5) $$\frac{dg_n(\omega)}{d\omega} > 0.$$

That is, one has the typical, increasing natural growth rate case, which is central to Morishima=Catephores.

The real wage rate at which the number of workers is stationary is called the subsistence real wage rate: write

ω_s: subsistence real wage rate,

(6) $$\omega_s = \{\omega | g_n(\omega) = 0\},$$

Note that the positivity of the subsistence real wage rate is guaranteed by (1) through (4).

Now, it must be observed that the decreasing natural growth rate case is also possible.

That the function $g_n(\omega)$ is increasing depends on (4). However, apart from rather irregular increases in the real wage rate concomitant with such as overtime jobs, it is empirically observed that $f(\omega)$ becomes decreasing even within the normal range of the real wage rate, if the real wage rate is sufficiently high: "diligence in poverty and idleness in prosperity" is also a possible case in capitalism.[4]

Needless to say, the constant natural growth rate case can be also considered.

Accordingly, the following three cases of the natural growth rate should be considered:

(I) the increasing natural growth rate case,

(II) the decreasing natural growth rate case, and

(III) the constant natural growth rate case.[5]

The following Figures 1 and 2 show the typical graphs of $f(\omega)$ and corresponding trapezium-shaped $g_n(\omega)$ dealt with in the next section.

It is easy to see that the graphs are a modification of original Morishima=Catephores' graph: one has only to bend the rightmost end of their graph.

4) This may be explained partly by the fact that if a member of a family gets high wages, other members of the family, especially housewives, need not to get jobs.

A decrease in the birth rate is also a great cause to lower the population growth rate. As a result of women getting jobs, the birth rate is decreasing in advanced capitalist countries.

5) Morishima=Catephores argued various possible patterns of the populaiton growth frontier: see pp. 127–41.

What is distinguished in this paper is the three typical phases of natural growth, and nothing beyond.

It should be remarked that the above three features of the graph concern the local nature of the graph.

Figure 1

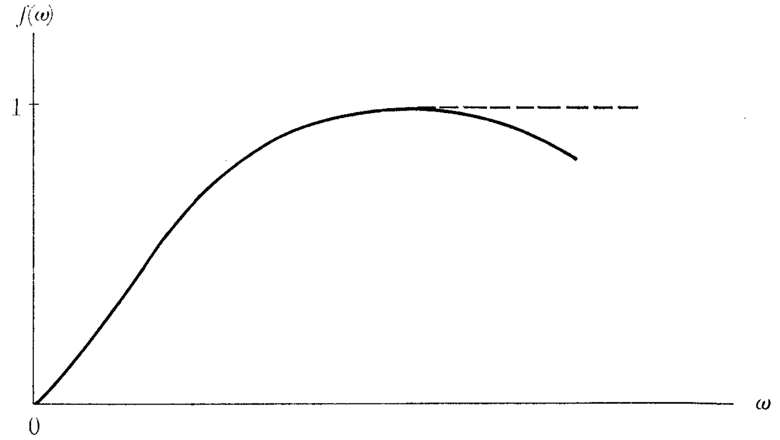
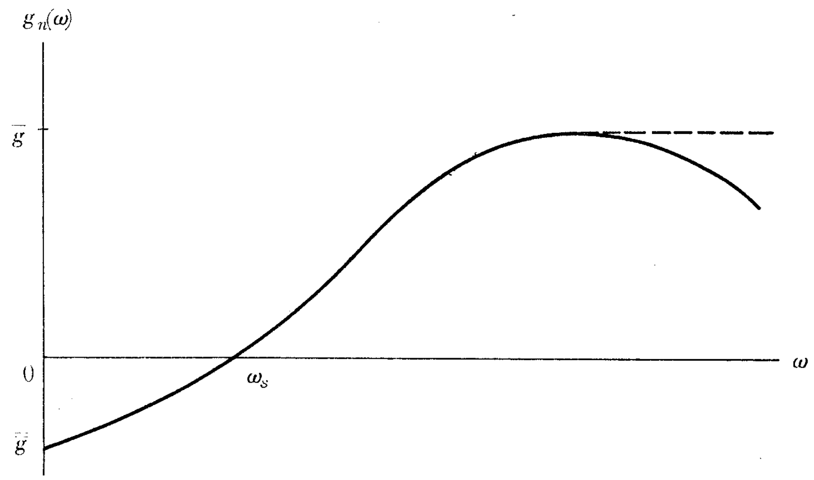


Figure 2



(Broken lines indicate original Morishima=Catephores' graphs.)

2. Assume that the capitalists do not consume and that the workers do not save. Then, the warranted growth rate is equalised to the profit rate. Let

$g_w(\omega)$: warranted growth rate,

and this is determined by

(7) $$g_w(\omega)=\max\{g|x=(1+g)(A+F(\omega)L)x,\ x\geq 0^n,\ pF(\omega)=\omega\}$$

where

$$p=(1+\pi)(pA+\omega L).$$

Now, one can define

DEFINITION 1 (Harrodian equilibrium) The state of the economy in which the equality

(8) $$g_w(\omega)=g_n(\omega)$$

holds is designated as a Harrodian equilibrium.

Let

g^*: Harrodian equilibrium growth rate,

ω^*: Harrodian equilibrium real wage rate,

x^*: Harrodian equilibrium output vector,

where

$$g^*=g_w^*(\omega^*)=g_n^*(\omega^*),$$

and (ω^*, g^*) is called a *Harrodian equilibrium point.*

A Harrodian equilibrium is said to be essential if $\omega^*, g^*>0$.

Let us assume:

(A. 1) $A\geq O,\ L\geq 0_n,\ F(\omega)\geq 0^n$.

(A. 2) $A+F(\omega)L$ is indecomposable,

and soon follows the well-known, continuous and decreasing wage-profit trade-off:

(9) $$\frac{dg_w(\omega)}{d\omega}<0.$$

Let

ω_M: maximum real wage rate,

and this is defined by

$$\omega_M=\{\omega|g_w(\omega)=0\}.$$

Morishima=Catephores showed the following:

PROPOSITION 1. Assume (4). A necessary and sufficient condition for an essential Harrodian equilibrium to exist is that

(9) $$\omega_s<\omega_M.$$

In fact, let us tentatively put

$$q(\omega)=g_w(\omega)-g_n(\omega),$$

and it is easy to see that

$$\frac{dq}{d\omega}=\frac{dg_w}{d\omega}-\frac{dg_n}{d\omega}<0.$$

If $\omega_s<\omega_M$, then

$$q(\omega_s)=g_w(\omega_s)>0,$$
$$q(\omega_M)=-g_n(\omega_M)<0,$$

and there exists ω^* such that $q(\omega^*)=0$, $\omega_s<\omega^*<\omega_M$ in view of Lemma.

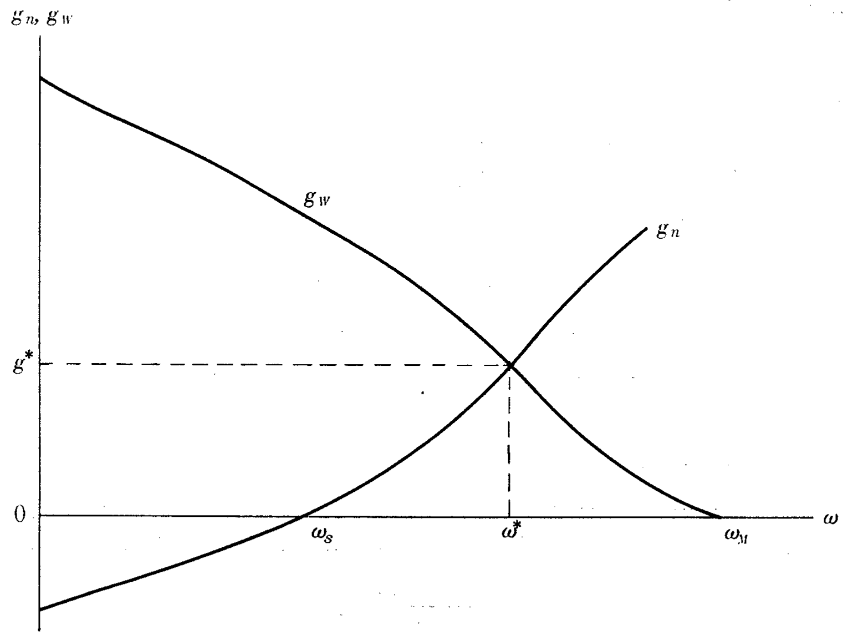
Contrarily, suppose $q(\omega)$ has a ω^* such that $q(\omega^*)=0$, then there exist ω_1 and ω_2 satisfying

$$q(\omega_1)=g_w(\omega_1),$$
$$q(\omega_2)=-g_n(\omega_2).$$

It is easy to see that $\omega_1=\omega_s$ and $\omega_2=\omega_M$ and that $\omega_s<\omega_M$.

This fact will be well illustrated by the following Figure 3.[6]

Figure 3



It should be remarked that (9) implies

(10) $$\rho[A]<1,$$

but not converse: (9) is stronger than (10).

3. Morishima=Catephores measured the expenditure of labour in terms of the number of workers multiplied by the length of the workday, and supposed that the length of the workday is a decreasing function of the real wage rate.

Let

T : length of the workday,

Σ : amount of labour,

and they wrote:

(11) $$\Sigma=NT, \quad T=\frac{1}{\omega}$$

It is easily seen that in their formulation the natural rate of growth may not be well represented by the growth rate of population if such a dynamic change with an increase or decrease in the real wage rate takes place.

Hence, in what follows, in order to facilitate a clear understanding of the analysis, the

6) Since the modified natural growth frontier is discussed, Proposition 1 applies to it.
If a globally decreasing or constant natural growth frontier is contemplated, one should have different conditions for an essential Harrodian equilibrium to exist.
If $g_n(\omega)$ takes a constant value, $g_w(0)>g_n(\omega)>0$ is necessary and sufficient; if $g_n(\omega)$ is decreasing, then $g_w(0)>g_n(\omega)$ and (10) are sufficient, but not necessary.

length of the workday is assumed to be given.

§ 2. Harrodian cquilibrium and technical changes

1. In the subsequent argument, an essential Harrodian equilibrium is treated: (9) is assumed to hold.

Suppose the economy is in dynamic growth with continuous technical changes. In period t, one can write:

$$M_t = A_t + F_t(\omega) L_t, \tag{12}$$

and by substituting this into (7), one can at once obtain the warranted growth curve of period t. A set of such growth curves $g_w(\omega; t)$ makes up the class of the warranted growth curves.

Let us call $g_p(\omega)$ the *(basic) population frontier.* The intersection of $g_w(\omega; t)$ and $g_p(\omega)$ constitutes the *stationary Harrodian equilibrium* of period t.

What should not be overlooked is that the population frontier will be shifted if an increase in labour productivity is brought about by technical changes. Such a shift will form the class of the natural growth frontiers, $g_n(\omega; t)$. The intersection of the warranted growth curve and natural growth frontier of period t is designated as the *dynamic Harrodian equilibrium* of period t.

Let

x^t $n \times 1$: balanced growth path,

α^t $1 \times n$: capital saving factor,

γ^t $1 \times n$: labour saving factor,

δ^t : growth rate of labour productivity,

and the dynamic growth process is assumed to develop as follows.

At period t, M_t is used, and at the end of the period, i. e., at the beginning of period $t+1$, technical changes represented by α^t and γ^t occur momentarily and without cost. Since production is carried out on the basis of M_{t+1} in the next period, the accumulation of capital is also carried out on the basis of M_{t+1} at the beginning of period $t+1$. The warranted growth rate of period $t+1$ should be equalised to its natural growth rate, which equals the population growth rate added by the increment owing to increases in labour productivity; thus, the real wage rate of period $t+1$ is determined.

In each period, balanced growth takes place: from period t to $t+1$, production is switched from x^t to x^{t+1}.

The economy continues to grow in the same manner, and its growth path can be traced by the locus of Harrodian equilibria. What will be argued here is this locus thus obtained.

2. Technical changes are not necessarily input-saving: the economy may become more profitable, for instance, by introducing new kinds of capital good, and hence augumenting the employment of capital. Nevertheless, the economisation of input of production or factors

is fundamental for technical changes.[7]

Since it suffices to examine in advance a single step of a technical change, say from period t to $t+1$, let us compare the two states of the economy. Write

(13) $$M_t=\hat{\alpha}_t A_{t-1}+F_{t-1}(\omega)L_{t-1}\hat{\gamma}_t.$$ [8]

In view of Frobenius' theorem, one soon gets:

PROPOSITION 2. If $\hat{\alpha}_t \geq I$ and $\hat{\gamma}_t \geq I$, then

$$\rho[M_t] \geq \rho[M_{t-1}].$$

That is, if there is no economisation of factors, the warranted growth rate can never be increased, which is fundamentally against the profit-maximising capitalists.

In what follows, it is assumed that

(A. 3) $$\hat{\alpha}_t \leq I,\ \hat{\gamma}_t \leq I.$$

3. First consider the case that α_t and γ_t are both scalars. Three cases can be distinguished.

(i) If $\alpha_t < \gamma_t < 1$, then it is called capital-saving.

(ii) If $\gamma_t < \alpha_t < 1$, then it is termed labour-saving.

(iii) If $\alpha_t = \gamma_t$, then it is neutral.[9]

Now, the increase in the warranted growth rate is evaluated as

(14) $$\Delta g_w(\omega\,;t)=g_w(\omega\,;t+1)-g_w(\omega\,;t),$$

where, needless to say,

$$g_w(\omega\,;t)=\frac{1}{\rho[M_t]}-1.$$

A difficulty may arise in evaluating the natural growth rate with technical changes in a microscopic framework, because the balanced growth path undergoes changes from period to peirod owing to technical changes, which makes it difficult to evaluate the industrywise increase of production potentiality.

Therefore, let us define the natural growth rate in terms of macro-level production potentiality.

DEFINITION 2. (*Natural growth rate*) The natural growth rate is measured by the effect of increases in employment possibly brought about by a technical change and production levels associated with it:

7) Marx referred to insistent economisation of caonstant capital (=capital goods) made by the capitalists. (III, Ch. V.)

8) According to the RAS method dealing with technical changes in input-output tables, this can be more generally represented by

(*) $$M(\alpha_1, \alpha_2, \gamma)=\hat{\alpha}_1 A\hat{\alpha}_2+F(\omega)L\hat{\gamma},$$

where α_1 and α_2 represent respectively factorwise and industrywise changes of capital goods, and usually $\alpha_2=\gamma$.

The following argument, however, does not undergo any alteration even if (*) is simply written as (13).

As for the RAS method, refer to Stone=Brown.

9) Cf. Heertje et al, p. 274.

(15) $$g_n(\omega\,;t+1)=\frac{L_t x^{t+1}}{L_t x^t}-1.$$ [10]

Remark that one has

(16) $$g_p(\omega\,;t+1)=\frac{L_{t+1}x^{t+1}}{L_t x^t}-1.$$

Now, since one has here

(17) $$L_t=L_{t-1}\gamma_t,$$

an increment in the natural growth rate over the population growth rate can be represented by

(18) $$\Delta\, g_n(\omega\,;t)=g_n(\omega\,;t+1)-g_p(\omega).$$

In view of (17), one gets

(19) $$\delta_t=\frac{1}{\gamma_t}-1,$$

so that (18) can be rewritten as

(20) $$\Delta\, g_n(\omega\,;t)=\delta_t(1+g_p(\omega)).$$ [11]

Compare (15) with (18), and one can show:

PROPOSITION 3.

(21) $$\Delta g_w(\omega\,;t)\gtreqless\Delta g_n(\omega\,;t)\quad\text{according as}\quad\alpha_t\gtreqless\gamma_t.$$

In fact,

$$\sigma_t M_t\geqq M_{t+1}\geqq\tau_t M_t,$$

where $\sigma_t=\max(\alpha_t,\gamma_t)$, $\tau_t=\min(\alpha_t,\gamma_t)$. It then follows that

(22) $$\left[\frac{1}{\sigma_t}-1\right](1+g_p(\omega\,;t))\leqq\Delta g_w(\omega\,;t+1)\leqq\left[\frac{1}{\tau_t}-1\right](1+g_p(\omega\,;t)).$$

As seen from the comparison of (14) and (18), the past technical changes are all cumulated in the shifts of the warranted growth curves: this may be called the *integral effect* of technical changes on the warranted growth rate.

Whilst, the shifts of the natural growth frontier are caused by momentary changes in labour productivity from one period to the next. This may be termed the *differential effect* of tehcnical changes on the natural growth rate.

4. Let S_t and D_t denote the stationary and dynamic Harrodian equilibria of period t respectively, and write as

$$S_t=(\omega_t^*,\ g_t^*),\quad D_t=(\omega_t^{**},g_t^{**}),$$

where "*" and "**" indicate respectively values of variables in the stationary and dynamic Harrodian equilibria.

Now, suppose that the economy grows with technical changes, and it is either in

---

10) It is natural that the effect of increases brought about by technical changes should be measured in an old system of techniques.

11) If $\delta_t g_p(\omega)$ is sufficiently small, $\Delta g_n(t)$ is reduced to δ_t: namely,

$$g_n(\omega\,;t+1)\doteqdot\delta_t+g_p(\omega\,;t+1).$$

stationary or dynamic Harrodian equilibria. Proposition 3 gives a clue to compare the locations of $\{D_t\}$ and $\{S_t\}$.

First, consider the case (iii).

PROPOSITION 4. If technical changes are neutral and $\alpha_t<1$, then:

(i) $$p^{t+1}=p^t \ ; \ x^{t+1}=x^t.$$

(ii) $$\omega^{**}_{t+1}=\omega^*_t \ ; \ g^{**}_{t+1}>g^*_t ,$$

and

$$g^{**}_{t+1}=g_w(\omega^* ; t+1)=g_n(\omega^* ; t+1),$$

(Irrespective of cases (I) through (III).)

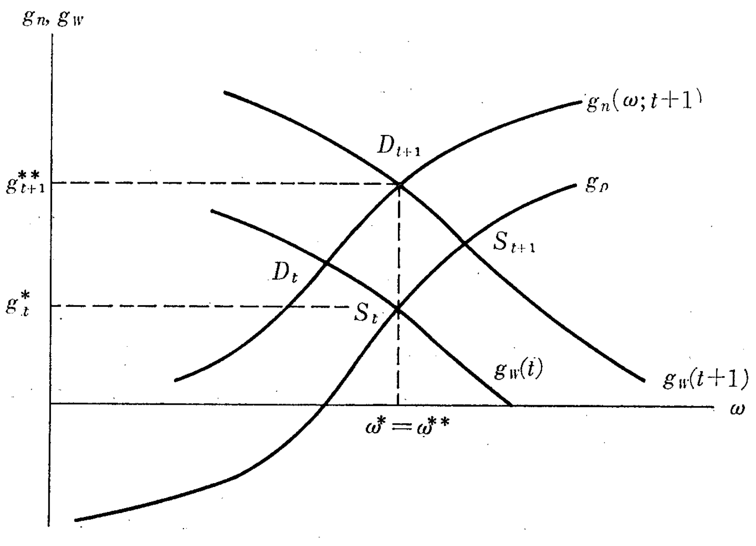
In fact, in the light of Frobenius' theorem, (i) is obvious. In view of Proposition 3, it soon follows that

$$\Delta g_w(\omega ; t+1)=\Delta g_n(\omega ; t+1)=\delta_{t+1}(1+g^*_t)$$

where $g^*_t=g_p(\omega^*)=g_w(\omega^* ; t)$.

This proposition is illustrated by the following Figure 4.

Figure 4.



Let us next contemplate the cases (i) and (ii).

PROPOSITION 5. Suppose the case (I).

(i) Capital-saving technical changes:

$$\omega^{**}_{t+1}>\omega^*_t \ ; \ g^{**}_{t+1}>g^*_t,$$

and

$$g_w(\omega^*_t ; t+1)>g^{**}_{t+1}>g_n(\omega^*_t ; t+1).$$

(ii) Labour-saving technical changes:

$$\omega^{**}_{t+1}<\omega^*_t \ ; \ g^{**}_{t+1}>g^*_t,$$

and

$$g_w(\omega_t^* ; t+1) < g_{t+1}^{**} < g_n(\omega_t^* ; t+1).$$

In fact, take (ii): since $g_w(\omega ; t+1)$ is decreasing and $g_n(\omega ; t+1)$ increasing with respect to ω, so that holds

$$g_w(\omega_t^* ; t+1) < g_n(\omega_t^* ; t+1)$$

in view of Proposition 3. Then, there exists $\omega < \omega_t^*$ such that

$$g_w(\omega ; t+1) = g_n(\omega ; t+1).$$

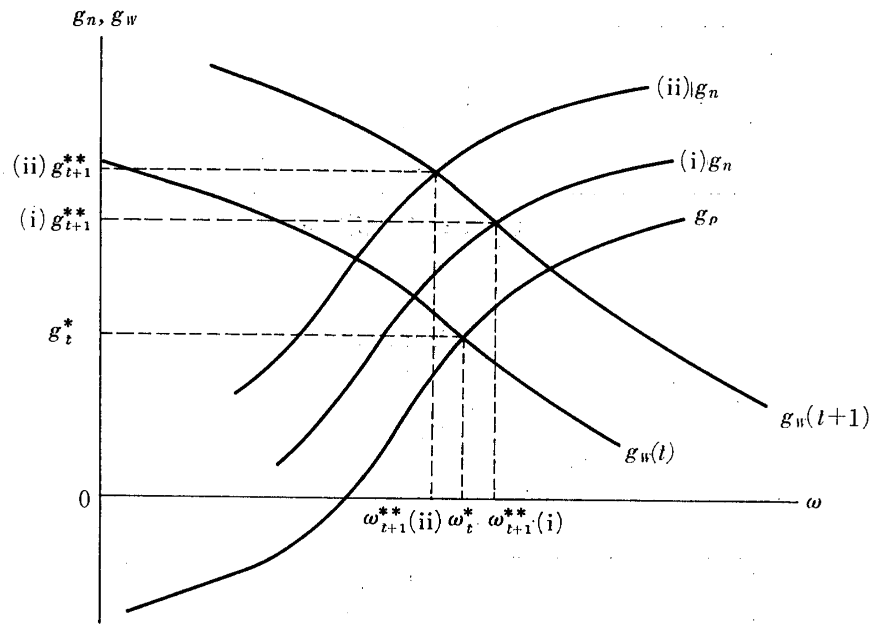
which defines ω_{t+1}^{**} for $g_w(\omega ;\quad)$ and $g_n(\omega)$ are continuous with respect to ω. Needless to say,

$$g_w(\omega_t^* ; t+1) < g_{t+1}^{**} = g_w(\omega_{t+1}^{**} ; t+1) = g_n(\omega_{t+1}^{**} ; t+1) < g_n(\omega_t^* ; t+1).$$

The case (i) will be shown in the same manner.

The above proposition is illustrated by the following Figure 5.

Figure 5.



5. As pointed in § 1, it is possible to consider the case that the warranted growth curves intersect the natural growth frontiers in a region where the latter are decreasing or constant.[12)]

PROPOSITION 5′. Suppose the cases (II) and (III).

(i) Capital-saving technical changes:

12) In those cases, the warranted growth curve may intersect the natural growth frontier more than once. However, what is argued here is the shift of Harrodian equilibria in a decreasing or constant range of the natural growth frontier. Hence, the multiple Harrodian equilibrium point case will not be treated here.

$$\omega^{**}_{t+1} > \omega^{*}_{t}\ ;\ g^{**}_{t+1} > g^{*}_{t},$$

and

$$g_w(\omega^{*}_{t}\ ;\ t+1) > g_n(\omega^{*}_{t}\ ;\ t+1) \underset{(=)}{>} g^{**}_{t+1}$$

(ii) Labour-saving technical changes :

Figure 6.

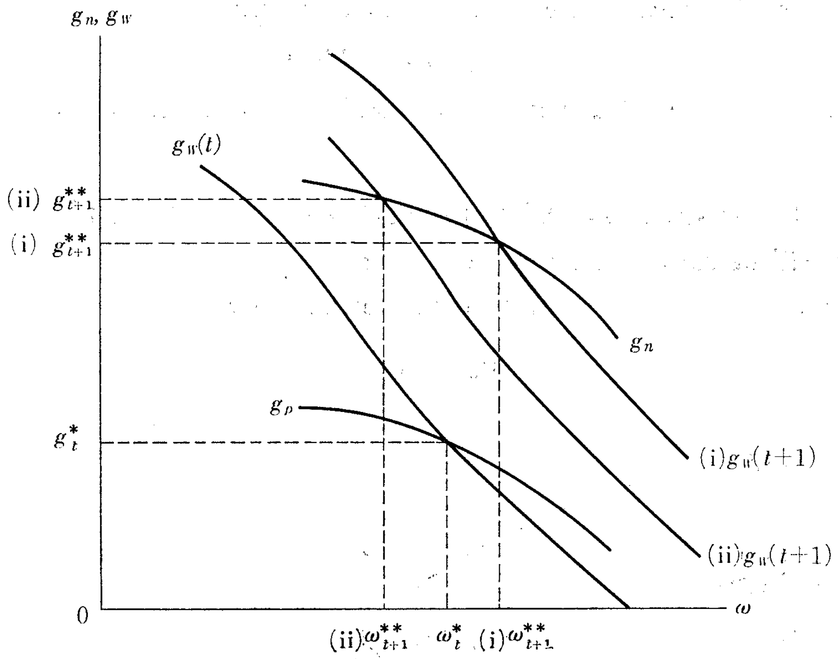
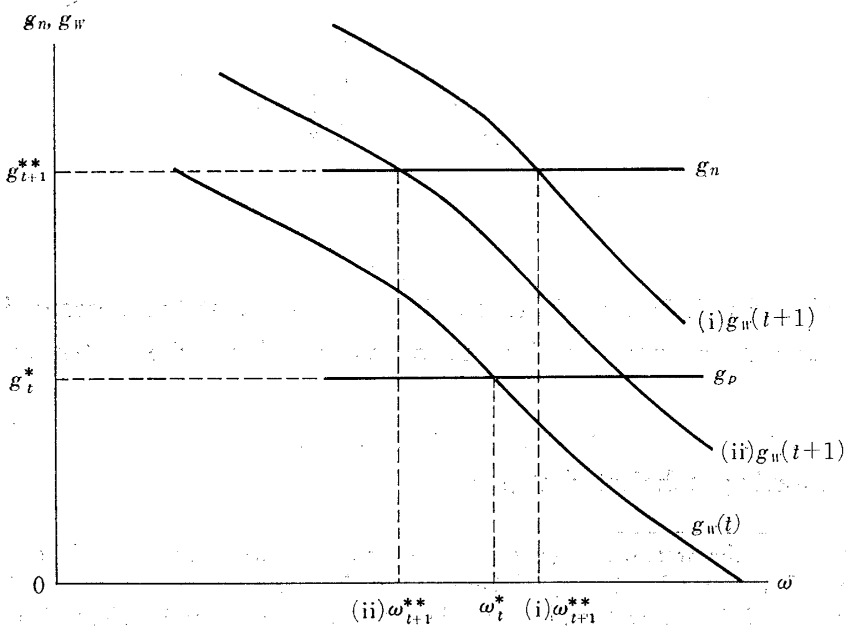


Figure 7.



$$\omega_{t+1}^{**} > \omega_t^* \ ; \ g_{t+1}^{**} > g_t^*,$$

and

$$g_{t+1}^{**} \underset{(=)}{>} g_n(\omega_t^* \ ; t+1) > g_w(\omega_t^* \ ; t+1).$$

(Equalities for the case (III).)

The above is illustrated by Figures 6 and 7. (See the preceding page.)

6. How will the preceding investigation be generalised if the industrywise or factorwise technical change is not uniform?

Regard, as originally presupposed, α_t and γ_t as vectors, and write

$$\alpha_t^M = \max \alpha_{tj}, \quad \alpha_t^m = \min \alpha_{tj},$$

$$\gamma_t^M = \max \gamma_{tj}, \quad \gamma_t^m = \min \gamma_{tj}.$$

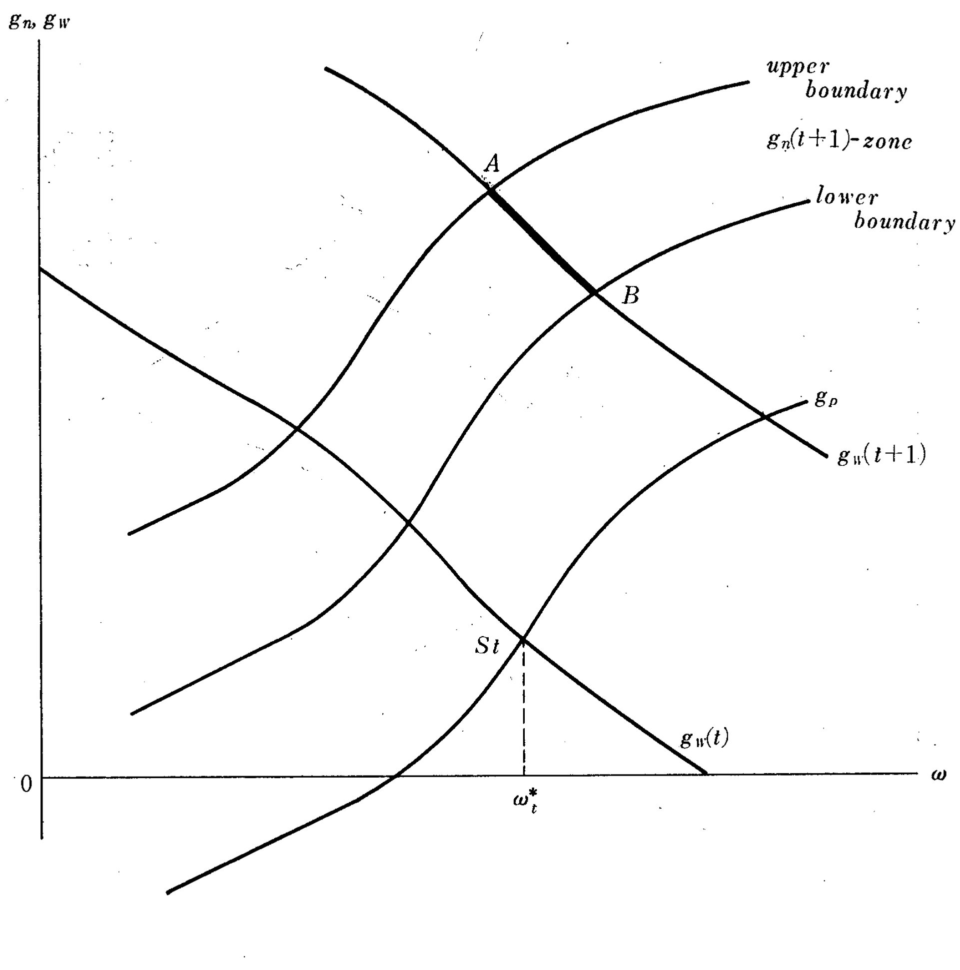
One has then

$$\sigma_t^M M_{t-1} \geqq M_t \geqq \tau_t^m M_{t-1},$$

so that

(23) $$\left[\frac{1}{\sigma_t^M} - 1\right](1+g_p(\omega)) \leqq \Delta g_w(\omega \ ; t-1) \leqq \left[\frac{1}{\tau_t^m} - 1\right](1+g_p(\omega)),$$

Figure 8.



if the following is true :

(24) $$\sigma_t^M \geqq \tau_t^M \ ; \ \sigma_t^m \geqq \tau_t^m,$$

where (i′) $(\sigma, \tau)=(\gamma, \alpha)$ or

(ii′) $(\sigma, \tau)=(\alpha, \gamma)$.

It is easy to see that (23) is an extension of (22) : if (i′) holds, an increase in the warranted growth rate is greater than the smallest industrywise increment in the natural growth rate, whilst if (ii′) holds, it cannot exceed the greatest industrywise increment.

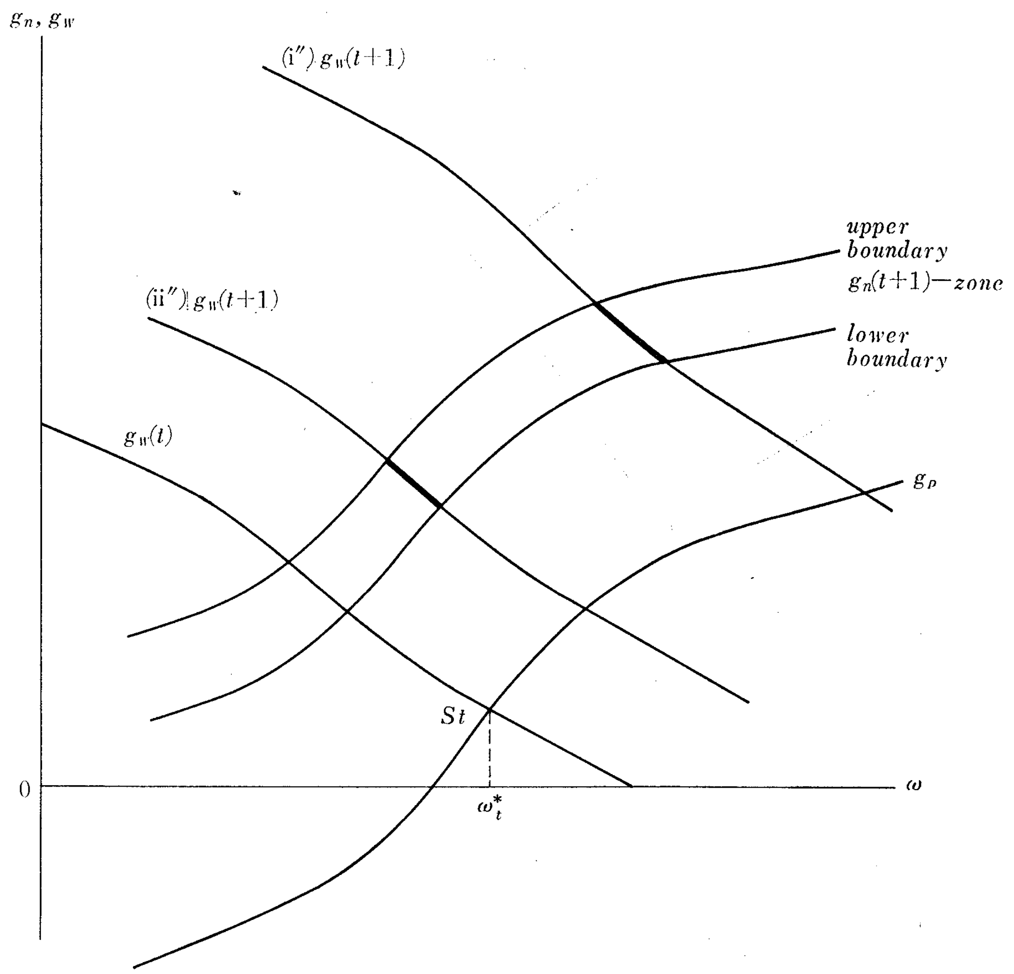
Let us call the natural growth curve corresponding to the smallest (or the greatest) industrywise increment in the natural growth rate the *lower* (or the *upper*) *boundary* of the natural growth frontier.

In the zone between these two boundaries, the natural growth frontier exists.

The following Figure 8 illustrates the case (I). Assume either (i′) or (ii′), and the Harrodian equilibrium point will be somewhere between A and B on the $g_w(\omega\,; t+1)$ curve.

Furthermore, if $\sigma^m > \tau^M$ is fullfilled, then (i′) will be called (i″) strongly capital-saving, and (ii′) becomes (ii″) strongly labour-saving.

Figure 9.



Supposing the case (I), one has the following Figure 9. (preceding page.)

Thus, instead of points, one can consider a dynamic Harrodian equilibrium region.

7. Let us briefly argue the case that the length of the workday is variable as posed by Morishima=Catephores: let us recall (11).

Now, since the population growth frontier represents N alone in this case, the natural growth frontier will be affected by the remaining two factors: the length of the workday and labour productivity.

In fact, one has if γ_t is uniform,

$$g_p(\omega\,;t+1)=\frac{N(\omega\,;t+1)-N(\omega\,;t)}{N(\omega\,;t)},$$

and

(25) $$\Delta g_n(\omega\,;t+1)=\{(1+\nu_{t+1})(1+\delta_{t+1})-1\}\,\{1+g_p(\omega)\},$$

in view of

$$L_t x^t=N(\omega\,;t)T(\omega\,;t),$$

where $1+\nu_{t+1}=T_{t+1}/T_t$.[13]

Let us call a natural growth frontier based on (11) a *modified* natural growth frontier. The intersection of a warranted growth curve and a modified natural growth frontier forms a *modified* dynamic Harrodian equilibrium, D_t'.

The following Figure 10 illustrates modified dynamic Harrodian equilibria and relative locations of stationary, dynamic and modified dynamic Harrodian equilibria of the case (I).

Let " ′ " indicate values of variables in modified dynamic Harrodian equilibria, and the subsequent facts can be confirmed:

PROPOSITION 6.

(i) Capital-saving technical changes:

$$\omega_{t+1}^{**'}>\omega_{t+1}^{**}\ ;\ g_{t+1}^{**'}<g_{t+1}^{**}.$$

(ii) Labour-saving technical changes:

$$\omega_{t+1}^{**'}<\omega_{t+1}^{**}\ ;\ g_{t+1}^{**'}>g_{t+1}^{**}.$$

Note that this proposition is valid for all cases (I) through (III). If a technical change is neutral, refer to Proposition 4.[14]

13) This can be derived as follows. Let Σ and N stand for vectors of the amount of labour and employment respectively, i-th component of which indicates industrywise magnitudes. Then, one can write

$$\begin{aligned}x^{t+1}&=\hat{L}_{t+1}^{-1}\Sigma\\&=T_{t+1}\hat{L}_{t+1}^{-1}N_{t+1},\end{aligned}$$

because $Lx=\Sigma=TN$.

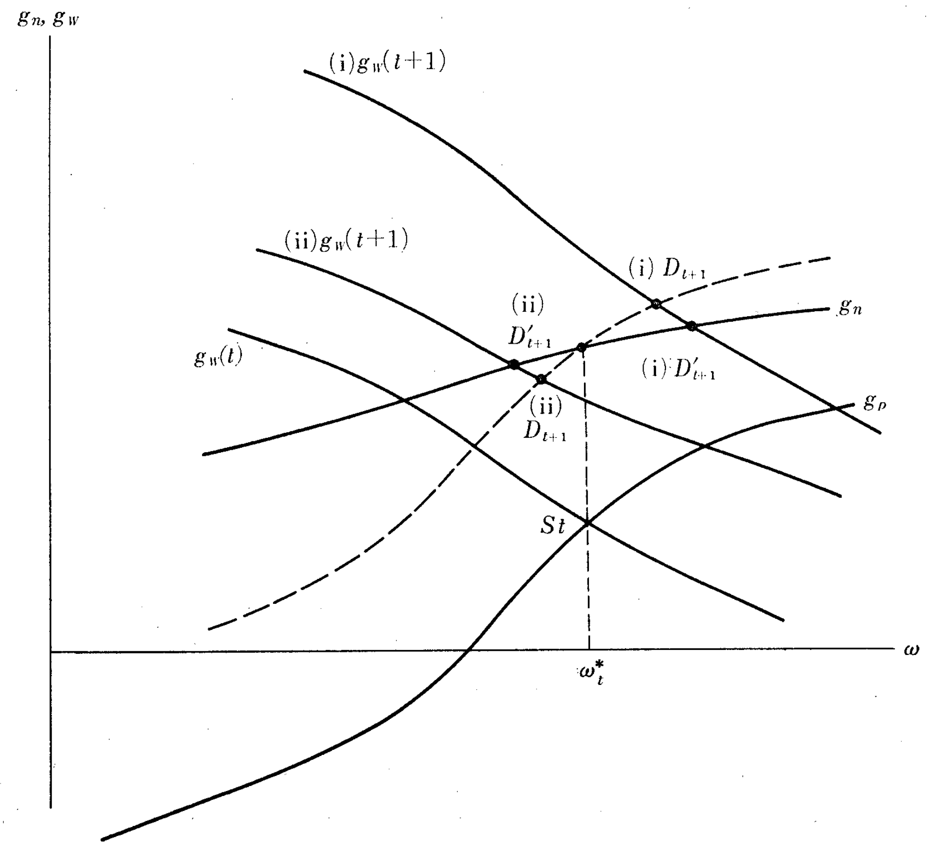
By putting those into (15) through (19), (20) can be rewritten as

$$g_n(\omega\,;t+1)=(1+\nu_{t+1})\,\{1+g_p(\omega)\}\,\frac{1}{\gamma_{t+1}}.$$

From this follows (25).

14) It should be observed that even if the population growth frontier is increasing the natural growth frontier may be constant or decreasing at a high real wage rate. This is another reason why the cases (II) and (III) are contemplated in this paper.

Figure 10.



(The broken line indicates the original natural growth frontier.)

8. What kind of effect does the economisation of factors bring about on Marx's value system?

Let

w $1\times n$: value vector,

ξ : organic composition of capital,

μ : rate of surplus value,

ν : rate of unemployment,

where value is determined by

$$w=wA+L.$$

Then, it is easy to see that (A. 3) entails $w^{t+1}<w^t$, so that

$$w^{t+1}A_{t+1}L_{t+1}^{-1}<w_tA_tL_t^{-1}.$$

Namely, dead labour/living labour ratios are decreasing, which does not depend on whether (i) or (ii).

Morishima=Seton's equality is written as

$$g_t^{**}=\frac{\mu_t}{\xi_t+1}. \tag{26}$$

In so far as the economy grows with continuous technical changes which are traced by a northeastward locus of $\{D_t\}$, g_t^{**} is increasing whereas μ_t is decreasing unless choice of consumption is permitted.: therefore, ξ_t must be decreasingthen. Namely, in this case, the organic composition of capital in dynamic Harrodian equilibrium is also decreasing.

Moreover, in such Harrodian equilibria, the rate of unemployment defined by

$$v^{**}=1-f(\omega^{**})$$

is also decreasing.

These reconfirm an analytical possibility of a counterexample against Marx's theory of capitalism to fall.[15]

Note that the labour-saving case can apply to the case $A_{t+1}>A_t$, which may imply the increasing organic composition of capital.[16]

§ 3. Concluding remarks — some implications.

1. Heretofore, an effect of technical changes on the Harrodian equilibrium has been discussed. Some additional remarks should be made by way of conclusion.

In the outset, the feature of the Harrodian equilibrium must be confirmed.

The establishment of the Harrodian equilibrium is based on the two factors — the natural growth rate and the warranted growth rate. These two factors may be looked at with reference to the Marxian concepts of the productive force and the production relation: the natural growth rate concerns rather the productive force, whilst the warranted growth rate reflects the production relation.

It must be noted that the Harrodian equilibrium still rests on the market equilibrium: when the rate of willing workers is interpreted as the rate of employment, it is supposed that supply of labour is equal to its demand in the labour market. This implies that the last independent equation to be added may concern the supply=demand equality.

The Harrodian equilibrium permits unemployment. Although the rate of unemployment depends on the function $f(\omega)$, not all the workers may be employed in the Harrodian equilibrium.

Accordingly, it is seen that the concept of Harrodian equilibrium comprises Marxian, neoclassical and Keynesian factors.

2. In the next place, some implications of technical changes argued in the above should be mentioned.

It has been shown that the economisation of factors is at least favourable to the capitalists.

15) Heertje et at also discussed the unemployment problem in Marx's economics in a different context, and drew the same conclusion as Morishima (p. 163). Both, however, dealt with the constant real wage rate case, and not the Harrodian equilibiurm.

16) As for the increasing organic composition of capital case in the two-department Marx=Leontief economy, also see Heertje et al.

Although the stationary Harrodian equilibrium of one period is compared with the dynamic Harrodian equilibrium of the next period, it is seen that both Harrodian equilibrium growth rate and real wage rate go up if a technical change is capital-saving.

Whilst, in the labour-saving technical change case, it is not certain whether the Harrodian equilibirum real wage rate is heightened, though the Harrodian equilibrium growth rate is increased.

It is clear that once technical changes stop at period t, the economy will jump down from D_{t-1} to S_{t-1}: namely, the Harrodian equilibrium point of period t is reduced to the stationary Harrodian equilibrium point of period $t-1$. Then, the Harrodian equilibrium growth rate looks down, whereas the Harrodian equilibrium real wage rate is raised.

If a labour-saving technical change is brought about in an economy in stationary Harrodian equilibrium, the Harrodian equilibrium real wage rate is momentarily lowered in that period.

In so far as technical changes are factor-saving, the locus of the economy will consist of dynamic and stationary Harrodian equilibria. Nevertheless, no definite conclusion can be drawn more than above.

3. Needless to repeat, tehcnical changes are not factorsaving: the economisation of factors is often carried out by augmenting some other types of factor.

However, the process of continuous economisation of factors in a capitalist economy may not be smooth sailing.

Marx emphasized the increasing organic composition of capital, but his true intention should be found in the negation of such smooth continuous technical changes in a capitalist economy.

From the viewpoint of the productive force, the maximum profit rate reflects the development of the productive force, so that fundamentally the economisation of factors asserts itself in the long-run.

Since a combination of factors depends on their various physical and chemical natures, their economisation is limitted, unless basically new techniques are invented. New techniques often comprise the employment of new types of good, but in terms of the maximum profit rate the comparison of old and new systems of techniques can be made.

Apart from practical difficulties, why is such a factor-economising technical change regarded as nonsmooth sailing by Marx and most Marxian economists?

According to the Marxian concept of history, the development of the productive force is arrested by the production relation. Hence, it may be relevant to link the increasing organic composition of capital with the conflict between the productive force and the production relation.

The production price system reflects the production relation of capitalist economies, enabling the capitalists to appropriate surplus value created by the workers in the form of profit. The transformation of surplus value into profit is based on capital, which is private property of the capitalists.

What distinguishes technical changes in capitalist and other, say planned, economies most is the social character of factors — private property or not.

If capital goods are less and less employed in production lines, the production price system itself would be sublated: if $A=O$, then the production price system as such enabling the capitalists to appropriate profit on the basis of the commodity production would not hold, whereas the case $L=0_n$ is impossible in Marx's context. In Marx's scenario, the quantitative weight of capital goods ought to be material in forming the production price system.

What drives capitalists to choose less capital-saving or even capital-absorbing techniques may be the fear to lose the power of capital dominating profit.

In an ideal planned economy in which the production relation fits the existing level of the productive force, factor-saving technical changes are considered to take place more smoothly than in capitalist economies by Marxian economists.

Any way, a difficulty in selecting a factor-saving technical change, if ever there is, should be ascribed to social and institutional backgrounds, since techniques are techniques irrespective of the social form of production.

Consequently, the point should be made clearer from the above mentioned angle if the increasing organic composition of capital were not to be rejected only from the analytical point of view.

4. What kind of place should be given to the Harrodian equilibrum?

Although technical changes discussed here are of momentary nature, the locus of Harrodian equilibria can be regarded as the long run growth path of the economy.

Appendix

The following fact is made use of in the proof of Proposition 4. It is also useful for the similar discussion.

Lemma. Suppose $y=h(x) \in C^1$. If $h(x_1)<0<h(x_2)$ for $x_1<x_2$, then there exists at least one x such that $h(x)=0$ and $x \in [x_1, x_2]$.

References

Fujimori, Y. (1981)., *Modern Analysis of Value Theory,* Josai Univ.

Harrod, R. (1973)., *Economic Dynamics,* Macmillan.

Heertje, A. (1977)., *Economics and Technical Change,* Weidenfelt=Nicholson.

Heertje, A., Furth, D., Van Der Veen, R. J. (1978)., "On Marx's theory of unemployment," *Oxford Economic Papers* 30 (2), pp. 263–76.

Marx, K. (1977)., *Capital* III, Progress Publisher, Moskow.

Morishima, M. (1973)., *Marx's Economics,* Cambridge Univ. Press.

Morishima, M., Catephores, G. (1979)., *Value, Exploitation and growth,* McGrawHill.

Okishio, N. (1977)., "Notes on technical progress and capitalist society," *Cambridge Journal of Economics* 1 (2), pp. 93–100.

Pasinetti, L. L. (1974)., *Growth and Income Distribution,* Cambridge Univ. Press.
———— (1979)., *Lectures on the Theory of Production,* Macmillan.
Sraffa, P. (1951)., *Production of Commodities by Means of Commodities,* Cambridge Univ. Press.
Stone, R., Brown, J. A. C. (1962)., "A long-term growth model for the British economy," R. C. Geary ed., *Europe's Future in Figures,* Ch. 10., North=Holland.