An Essay on the Theory of Prices

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INTRODUCTION

1. The economic theory has been developed in line with the capitalist economy. Since the development of capitalist economies is to establish the price system that is specific to the capitalist economy, the development of economic theory has been concerned with price theory as ist core.

None will argue that the problem related to prices of goods have been one of the hreatest concerns of economics. It must be observed, however, that there have been various theories of prices. That is, economists do not share the same view as to fundamental problems of price: how prices are determined, how and why profit accrues to capital, etc.

The classical economics contemplated the natural price of commodities, and later Marx developed it to the concept of production price. The production price is related to a type of equilibrium, and hence is a kind of equilibrium price.

Another type of equilibrium price developed in the economic theory is the market equilibrium price: the optimum state of rational economic agents is created, if commodities are exchanged at their market equilibrium price. This theory has been developed by the neoclassical economic theory since Walras.

Price theory has been developed as a core of micro economics, but it seems that pricing of commodities is not discussed in the context of the economic circulation, or the reproduction process, to full extent. That is, the cycles of reproduction from production to consumption of an economic system should be repeatable, and one must inquire if the existing price theories can present a complete explanation concerning the cycle of reproduction.

The objective of this short paper is to investigate the nature of pricing in each step of the reproduction process.

When we discuss the reproduction process, we should not overlook the fact that it takes time for the reproduction process to develop. That is, the reproduction process has a specific time structure.

In the first place, we shall discuss the time structure of the reproduction process, and in the next place we shall establish the price systems correspoding to that time structure of the reproduction process.

2. The assumptions and the framework of analysis of the following discussion are fundamentally the same as the ones adopted by Fujimori(1982).

In order to discuss the pricing and the time structure of reproduction, however, it is necessary to investigate the implications of (# 3) and (# 4). They are:

(# 3) Each type of good is produced in a unit period.

(# 4) Each process is of point-input and point-output type.[1]

A cycle of reproduction, which develops from production to consumption via exchange and distribution, can be well diagramatized by the schema of circuits of capital which Marx presented in his *Capital* II.

Borrowing from Marx, we have the circuit of production capital:

$$C \begin{cases} \text{Pr.} \\ \text{L.P.} \end{cases} \cdots P \cdots C'—M'—C'' \begin{cases} \text{Pr.} \\ \text{L.P.} \end{cases}$$

(where Pr. and L.P. respectively stand fer the means of production and labour-power.)[2]

The essence of the circuit is boiled down to that exchange and production constitute the major parts of the reproduction process. From the viewpoint of "production of commodities by means of commodities," the major part of the reproduction process is the transformation of inputs into outputs and the allocation of outputs to inputs.

Let us take the circuit of production capital, and suppose the following usual situation. At the beginning of a period, the means of production. or capital goods, and labour-power are combined in production lines, and the commodities are produced in one period. Then, they are sold and purchased in the market in order to allocate them for the production of goods in the next period.

Now, assumption (# 3) means that for each of these three circuit of capital it takes one period to develop normally. Since the whole reproduction process develops in the same duration of time, this at the same time means that exchange of goods is carried out momentarily.

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1) The remaining assumptions are:

(# 1) The number of goods and the number of processes are finite.

(# 2) Techniques are linear.

(# 5) The population of society is resolved into two major classes, i.e., the capitalist class and the working class.

(# 6) The workers do not save.

(# 7) Choice in consumption is disregarded.

(# 8) Wages are paid in advance.

2) The two other circuits that Marx mentioned are:

(I) The circuit of money capital:

$$M—C \begin{cases} \text{Pr.} \\ \text{L.P.} \end{cases} \cdots P \cdots C'—M'$$

(II) The circuit of commodity capital:

$$C—M—C' \begin{cases} \text{Pr.} \\ \text{L.P.} \end{cases} \cdots P \cdots C''—M''$$

Momentary exchange seems to carry no actuality, but it implies that once supply and demand are equalised, they are synchronised, however long it may take for their adjustment. If a highly organised market is considered, supply and demand can be adjusted in a very short time.

Thus, it is seen that the fundamental time-structure of the reproduction process is, first, the time lag in production, and, second, timeless exchange.

3. An important point here is that production needs time: there is a time lag between inputs and outputs. Once this is confirmed, the following problem can be raised.

Suppose that the cycle of reproduction is in period t. Looking at the development of the reproduction process, we see that inputs for production activities exist at different time point, or date, with outputs of production. If things in general are distinguished by their dimension, location and date,[3] inputs and outputs may not be comparable immediately: t is not sufficient to specify the existence of things. Then, we can immediately see that the traditional way to evaluate net products, surplus products and profit mentioned in most of literature will encounter difficulty, if there is a time lag between inputs and outputs, and hence they have different dates.

Since prices are related to comparability of commodities, their determination is expressed in terms of equality or inequality. Two (or more) types of goods are compared to each other in price systems, but how are commodities made comparable to each other from the angle of their dates? How is it possible to evaluate profit when results of production have different dates with their inputs?[4]

4. It may make sense to remark the following in order to avoid a possible misunderstanding. The role of the time factor in economics has been dealt with by the Austraian school. Burmeister(1974) demonstrated that the traditional Austrian model can be reduced to a von Neuman model. Our focus of discussion here is, however, the difference in dates of inputs and outputs. This difference still remains even after reducing the Austrian model to the von Neumann model.

5. Based on the above, we shall develop the theory of production price and market price with respect to the time structure of the reproduction process.

CHAPTER I
Time Lag in Production and Production Price

§ 1. Dates of goods and the production price.

1. Let us first have a closer look at how the reproduction process is developed.

Given a unit period, called period t, taken from a series of periods, we attach subscript t to all the variables, both endogenous and exogenous, related to period t. We shall

3) Cf. Debreu, pp. 29–30.

4) An immediate justification is to suppose that production needs no time, but this is not our point.

identify the beginning of period t with the end of period $t-1$, but there will be no confusion.

At the end of period t, products come out of production lines, and at the same time point, they are allotted for production of goods, and in one period, at the end of period $t+1$, in the same way goods are allotted for various purposes.

Naturally, inputs of production of period t, Mx^t, should precede the outputs of that period, x^t. As mentioned earlier, inputs Mx^t and outputs x^t have different dates: this can be illustrated by the following:

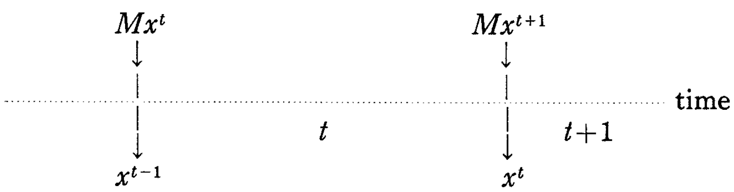

Figure 1.

Since inputs of a period is dated earlier than outputs of the period by exactly one unit length of period, it is easily seen that the input matrix M is a backward shift operator: if x^t is produced at the end of period t, then the amounts of inputs Mx^t should be laid out at the beginning of period t.

Therefore, without introducing a forward shift operator, we cannot immediately relate outputs to their inputs in establishing equations involving both inputs and outputs of a period. The same can apply to the labour input vector, but note that consumption is supposed to be done momentarily.

DEFINITION 1. 1. An operator which shifts the date of things from present to future, say from period t to $t+1$, is called the *time factor*.

In our following discussion, the time factor is often represented by a scalar.

Let us first take the quantity system. Let us write:

δ: time factor

Then, inputs and outputs are related to each other as:

(1) $$x^t = \delta M x^t.$$

This equation represents that the present goods, i.e., inputs Mx^t, is synchronised to future goods, i.e., outputs x^t. This can be rewritten as

(2) $$\frac{1}{\delta} x^t = M x^t,$$

which means that the future goods are discounted and synchronised to the present goods.

2. Let us now turn to the production price system. A closer look at the input-output relationship will be necessary.

As mentioned earlier, inputs of a production line are laid out, say at the beginning of period t, and, one period later, at the end of period t, outputs are available. As opposed to the quantity system in which a physical measure of goods can be regarded as independent of time, we have to distinguish prices of goods existing at the beginning of period t from those at the end of period t. Let us write:

$p^t \in {}^nR$: price vector of goods at the end of period t.

Then, at the beginning of period t, inputs of production, M, is evaluated as $p^{t-1}M$, and outputs to follow will be evaluated by p^t. Apply the time factor, and we can establish the production price system:

DEFINITION 1.2. A sequence of prices $\{p^t\}$ which synchronises the price of present goods to that of future goods is called the *dynamic production price*.

In the Leontief economy without technical changes, the above defined dynamic production price is determined by

(3) $$p^{t+1} = \delta p^t \cdot M,$$

The production price thus defined is referred to as dynamic production price, because it is of intertemporal nature.

Observe that the production price is based on the future techniques. This is becausea technique, which is regarded as normal one to produce one type of good, is not the technique that has been actually used, but the one that will be used to reproduce that type of good. In other words, the production price discussed here is the *re*production price.

3. Since the production price system (3) is a homogeneous diference equation system, it can be solved in the following manner.

Write

(4) $$M^+ = \delta M$$

and one obtains a particular solution:

(5) $$p^t = p^0 (M^+)^t,$$

where $p^0 \in {}^nR$ is arbitrary.

In order to see the nature of this, let us define here:

DEFINITION 1.3. $p^* \in {}^nR$ satisfying

(6) $$\rho[M] p^* = p^* M$$

is called the *static* production price, and

(7) $$\pi = \frac{1}{\rho} - 1$$

the *static* profit rate.

Let us assume as usual:

A1 $\quad A \geqq O, \quad F \geq 0^n, \quad L > 0_n.$

A2 $\quad A$ is indecomposable.

Then, we obtain the following:

PROPOSITION 1.1. Assume A1 and A2. Suppose thaf M is stable. Then

$$p^t \rightarrow \begin{cases} \infty \\ cp^* \\ O_n \end{cases} \quad (t \rightarrow \infty) \Longleftrightarrow \delta \gtreqqless 1 + \pi$$

where c is arbitrary but fixed.

In fact, the solution of (3) can be expressed by

(9) $$p^t = k_1 \rho_1{}^t \cdot p^* + \sum_2^n k_i \rho_i{}^t \cdot p^i,$$

where ρ_i and p^i are eigen values and their eigenvectors of M, ρ_1 being the greatest eigenvalue with p^* its eigenvector and $k_i s$ arbitrary constants $(i=1, \cdots, n)$.

That is, the particular solution $\rho[M^+]^t \cdot \theta[{}^tM^+]$ converges to 0_n or diverges to infinity unless $\rho[M^+]=1$. These two cases being not important, the profit rate added by unity must be equal to the magnitude of the time factor.

Moreover, we obtain:

PROPOSITON 1.2. Assume A1 and A2. Suppose that M is stable. Then, there exists $c>o$ such that

(10) $$\lim_{t\to\infty} p^t = cp^*.$$

(As for the proof, refer to Nikaido (1968, pp. 110–14.))

This proposition means that if M is nonnegative and stable, then the sequence $\{p^t\}$ satisfying (3) converges to a limit which is proportional to the static production price, even if $\delta \neq 1+\pi$.

4. The above provides an important clue to a deeper insight of the production price.

At the outset, the implication of the above is that if the condition of an economy represented by (I, M) is repeated ad infinitum, the dynamic production prices converges to the static production price. This may suggest that some kind of steady state needs be contemplated behind the establishment of the static production price.

In the second place, the above propositions deal with the convergence of a general solution of (3) to a particular solution (5), and whether or not the limit is economically meaningful depends on the magnitude of the time factor. Only the case where

(11) $$\delta = 1+\pi$$

provides an economically significant limit of the dynamic production price.

Since the determination of a static profit rate depends on techniques and wages, we can reconfirm that the equality (11) indicates the dependence of the time factor on the static profit rate: the time factor is another expression of the profit rate, and its magnitude must not be given a priori.

§ 2. Technical changes and a von Neumann economy.

1. Let us extend the previous discussion to the economy with technical changes. First, consider a Leontief economy. We shall apply t to A, F, L, M and δ so as to indicate period t.

No sooner are techniques assumed to be changing from period to period, than the static production price and the limit of the dynamic production price system are seen to be different in general.

In fact, the dynamic production price system is expressed by

(12) $$p^{t+1} = \delta_{t+1} p^t \cdot M_{t+1},$$

whereas the static production price system by

(13) $$p^{*t}=\delta_t p^{*t}\cdot M_t.$$

Write

(14) $$M_t^{+}=\delta_t M_t,$$

and (12) is rewritten as

(15) $$p^{t+1}=p^t\cdot M^{+}{}_{t+1},$$

and (13) as

(16) $$p^{*t}=p^{*t}\cdot M_t^{+}.$$

Given the sequence $\{M_t^{+}\}$, a general solution of (15) is given by

(17) $$p^t=C(t)\tilde{P}(t),$$

where $C(t)\in{}^nR$ is an arbitrary constant with cycle 1, i.e., $C(t)=C(t+1)$, and $\tilde{P}(t)$ is the fundamental matrix of the system.

On the other hands, its particular solution is given by

(18) $$p^t=p^0\prod_0^t M_k^{+}.$$

These solutions do not fulfill (13) in general.

2. In what case do the proportions of the two types of production price coincide with each other?

One of the simplest cases may be the one in which

(19) $$M_{t+1}=\alpha_t M_t,$$

where $\alpha_t\in R_+$. It is easy to confirm:

PROPOSITION 1.3. Assume A1. If (19) holds, then a particular dynamic production price $p^t=p^{*0}\prod_0^t M_k^{+}$ is proportional to the static production price: $p^t\propto p^{*t}$.

In fact, by taking (19) into account, (14) and (15) are rewritten respectively as

(20) $$p^{t+1}=\delta_{t+1}p^t(\alpha_0\cdots\alpha_{t+1})M_0^{+}$$

and

(21) $$p^{*t}=p^{*t}(\alpha_0\cdots\alpha_t)M_0^{+}.$$

Since we have $p^t=p^{*0}(\alpha_0\cdots\alpha_t)(M_0^{+})^t\propto p^{*0}$, it soon follows that

$$p^t\propto p^{*0}=p^{*t}.$$

In general cases, however, this proportionality will not be observed.

Now, can we say something about the stability of a particular solution (18)?

Although it becomes difficult to find a production price equilibrium when techniques are changing from period to period, the particular solution can be regarded to be stable in the following sense. Let us define:

DEFINITION 1.4. A scalar π^{+} and a vector $p^{+}\in{}^nR$ satisfying

$$p^{+}=(1+\pi^{+})p^{+}C,$$

where $C^{n+1}=M_0M_1\cdots M_n$, are called respectively the *average static profit rate* and the *average* *static production price*. A pair of them may be referred to as the *average system* of an economy with changing techniques.[1]

Then, we may say that the sequence of prices $\{p^t\}$ is converging to p^+, because under the same assumptions as stated in Proposition 1.2. $p^t = p^0 C^{n+1}$ can be regarded as a particular solution of the dynamic production price system $p^{t+1} = p^t \cdot \delta^+ C$, where $\delta^+ = (\delta_0 \delta_1 \cdots \delta_t)^{1/(n+1)}$, the average time factor.

3. Let us next consider a von Neumann economy.

Given the sequence $\{(B_t, M_t)\}$, the dynamic von Neumann price equilibrium (δ_t, p^t) is determined by

(23) $$\text{Min } \{\delta_{t+1} | p^{t+1} B_{t+1} \leq \delta_{t+1} p^t \cdot M_{t+1};\ p^{t+1}, p^t \geq 0_n\},$$

whereas the static von Neumann price equilibrium (δ_t, p^{*t}) by

(24) $$\text{Max } \{\delta_t | p^t \cdot B_t \leq \delta_t p^t \cdot M_t,\ p^t \geq 0_n\}.$$

The following proposition is immediately obtained:

PROPOSITION 1.5. i) If the dynamic von Neumann price equilibrium satisfying $p^{t+1} > p^t$ exists, then the static von Neumann price equilibrium fulfilling (24) also exists. ii) If the static von Neumann price equilibrium exists, then there exists a dynamic von Neumann price equilibrium with $p^{t+1} < p^t$.

In fact, as for i), from (23) one has immediately

$$p^{t+1} B_{t+1} \leq \delta_{t+1} p^t \cdot M_{t+1} \leq \delta_{t+1} p^{t+1} M_{t+1},$$

so that the conclusion soon follows.

Remark that the dynamic production price may be rather fundamental to the static production price. Nothing further, however, can be said in the above generalised case.

§ 3. Some concluding remarks.

1. The pricing associated with production, that involves time lag, yields two types of production price—the dynamic and static production prices. They depend on the time factor which equalises inputs and outputs with different dates. If a state of an economy is regarded as repeating itself ad infinitum, the dynamic production price will converge to the static production price. That is, the static production price can be looked at as a stable limit of the dynamic production price. In order for the limit to make sense, the magnitude of the time factor should not be given a priori. It is determined on the basis of techniques and wages.

Although the dynamic production price may appear less solid than the static production price, the former is more fundamental than the latter from the standpoint of dating things, because the former describes things as they stand on the time axis.

2. The condition that the state of an economy is repeatable ad infinitum suggests that the establishment of the production price is related to a type of steady state. Any way, it has something to do with the actual state of the economy. As indicated by the expectation of future prices, this implies that the production price is related to the producer's behaviour. Behavioural aspects, however, will not be treated here.

1) It is not always possible to find C for a given sequence $\{M_n\}$. This will be discussed in other place.

CHAPTER II.
Market Equilibrium Price

Introduction

The production price so far discussed concerns literaly the production process—it has been discussed how inputs are transformed into outputs through production. Our next task is to discuss how outputs are transformed into inputs, that is, the latter half of the circuit of production capital.

Since in the reproduction process the production process and the exchange process are presupposed to develop alternatively, exchange itself can be purely discussed, independent of production.

In relation to the circuit of capital, the market is defined as a field in which $C—M—C'$ takes place. It is a field of pure exchange. The market price will be defined as the price associated with this pure exchange.

In considering the market, the concept of supply and demand and their dependence on market prices are indispensable. In referring to exchange in the market, we presuppose atomistic agents. By atomistic agents we mean that their balance account is independent, they make decisions concerning production by themselves and that there is no coalition among them. It also means that each agent has to procure necessary goods by selling his own goods, whether or not his initial endowment contains money.

In what follows we assume spot transactions, so that forward exchange of goods is excluded. Moreover, we add

(# 1a) The number of entreprenurs is finite.

Here, the entreprenur is defined as the subject of production, whether humanor not, which is often called the capitalist.

§ 1. Market price

Let us recall the time structure of production illustrated by Figure 1 in Chapter I.

As mentioned in the introduction of Chapter I, if production of goods needs exactly one period, the exchange process is carried on momentarily. This may appear peculiar, but it implies that exchange of goods is synchronised: however long it may take to adjust supply and demand, exchange is synchronised, once transactions are made. Therefore, the timeless exchange hypothesis is sufficient for our present objective to investigate the relationship between pricing and the reproduction process.

Let there be k agents indexed by $Z(k)=\{1\cdots k\}$. Let s_{ij} and d_{ij} stand for supply and demand of good j by agent i. The equilibrium condition in the market is written in physical terms as: for all $j\in Z(n)$

(1) $$\sum_i s_{ij}=\sum_i d_{ij}.$$

This equality appears to be independent of any price of goods. In fact, let v_j be a valuation of good j, and we have

$$v_j \sum_i s_{ij} = v_j \sum_i d_{ij}, \tag{1'}$$

which is equivalent to (1) for any nonzero v_j.

We have to recall, however, that the market is the field where atomistic agents announce their supply and demand. For atomistic agents, it does not matter whether or not the total supply is equal to the total demand. They evaluate, however, their supplies and demands from their own standpoint.

DEFINITION 2.1. The *gross revenue* of agent i is expressed by $\sum_j v_j s_{ij}$ and the *gross outgoing* by $\sum_j v_j d_{ij}$.

From the standpoint of atomistic agents, the gross revenue must be sufficient to finance the latter. Here, we can define the market price:

DEFINITION 2.2. The amount of gross revenue of agent i subtracted by his gross outgoing is called the *surplus revenue* of agent i: for $v \in {}^nR$

$$\varepsilon_i = \sum_j v_j s_{ij} - \sum_j v_j d_{ij}. \tag{2}$$

DEFINITION 2.3. A nonnegative valuation of goods, $v \in {}^nR_+$ at which surplus revenue of all agents is zero: for all $i \in Z(k)$

$$\varepsilon_i = 0 \tag{3}$$

is called the *market price* of goods.

That is, for each agent in the market, the difference between sales and purchase appears to be a pure profit of exchange, and the market price is the price at which each agent can obtain his necessaries by his sales alone. In this sense, the market price reflects the subjectiveness of agents as well as the supply-demand adjustment.

In what follows, we shall consider a Leontief economy and discuss market price in detail.

§ 2. Market equilibrium price in a Leontief economy.

1. In general, agents in the market involve workers, but if we presuppose that wage goods are immediately paid from the hand of the capitalist class, we may identify all the agents with capitalists. This does not invalidate the following analysis.

Assume that capitalist i produces good i. Let us denote:

$p^t \in {}^nR$	:	market price vector,
$U_t^j \in R^n$	:	luxury goods bundle for capitalist j,
$U_t = (U_t^1, \cdots, U_t^k)$	:	luxury goods matrix,
$u^t = U_t \cdot 1^n$	:	luxury goods vector.

At the end of period t, the supply of goods is represented by x^t, whereas the demand of goods by $Mx^{t+1} + u^t$.

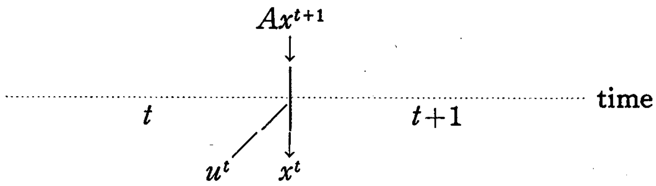

Figure 2.

Since demand and supply in the market are synchronised, the market price of period t should satisfy

(4) $$\bar{p}^t\cdot\hat{x}^t=\bar{p}^t(M\cdot\hat{x}^{t+1}+U_t).$$

First, suppose that there is no capitalist consumption, $u^t=0^n$, and that the economy is in steady growth: the quantity system is expressed by

(5) $$x^t=(1+g)Mx^t.$$

Then, the market price of this case is determined by

(6) $$p^t=(1+g)p^t\cdot M.$$

That is, the market price is equal to the static production price, if there is no capitalist consumption and the economy is in steady growth.

Nextly, suppose that capitalist consumption is not zero, but that the economy is again in steady growth. Since part of gross revenue is expended for luxury goods, we have to define the rate of saving.

DEFIRITION 2.4. The *market profit rate* is defined by, for $v\in{}^nR$,

(7) $$r_j=\frac{v_j}{v\cdot M^j}-1.$$

DEFINITION 2.5. The *rate of saving* of capitalist j is given by: for $p^t\in{}^nR_+$,

(8) $$\alpha_j(p^t)=\frac{r_j\cdot p^t\cdot M^j-p^t\cdot U_t{}^j}{r_jp^t\cdot M^j}.$$

DEFINITION 2.6. Market price, $p^t\in{}^nR_+$, is said to be *market equilibrium price,* if and only if for all $i, j\in Z(n)$

(9) $$r_i=r_j.$$

We have two important remarks here. The first remark is that the concept of profit in the above is different from that in the definition of the production price system, because the market profit rate does not carry any meaning of the time factor. Even additional capital investment appears to be a part of cost for future production, so that profit is obscured in the market. The "true" profit for capitalists in the market is the suplus revenue. The second remark is that in discussing the equilibrium market price itself, the adjustment process needs not be dealt with. We discuss the existence of the market equilibrium price, and not its stability. If the market equilibrium price exists, it is sufficient for the following analysis.

2. Now, we can rewrite (5) as

(10) $$\bar{p}^t=\bar{p}^t\cdot M(I+\hat{r}),$$

where

(11) $$\hat{r}=\hat{g}\cdot\hat{\alpha}^{-1}.$$

It soon follows:

PROPOSITION 2.1. Assume A1—A2. If for all $i, j \in Z(n)$ $g_i = g_j > o$ and $\alpha_i = \alpha_j > o$, then there exits a $\bar{p}^{-t} > 0_n$ such that (10) holds. Moreover, $\bar{p}^t = p^{*t}$, and $r = \pi > o$.

Proof.

If $g_i = g_j$ and $\alpha_i = \alpha_j$ for all $i, j \in Z(n)$, then (10) can be rewritten as

$$\bar{p}^t = (1+r)\bar{p}^t \cdot M,$$

whereas we have from the discussion in § 1 of Chapter I

$$p^{*t} = (1+\pi)p^{*t} \cdot M.$$

In view of Frobenius' theorem, we have $\pi = r > o$, and $\bar{p}^t = p^{*t}$. q.e.d.

It must be noted that, as shown by (11), the market profit rate depends on the growth rate of capital, namely the market profit rate depends on the subjective accumulation behaviour of capitalists.

Since each capitalist attains the market profit rate by his subjective accumulation policy, the competition among capitalist will eventually given rise to equal growth rates and saving ratios, which are represented by the presuppositions of the above proposition. If they are met, then the market price is equal to the static production price. That is, the so-called Cambridge equation is a sufficient condition for capitalists to find the static production price in the market. This is one of the remarkable features of a Leontief economy.

It is easy to see that even if there is a technical change from period to period, we can establish the similar proposition if we take the static production price, because the same augmented input matrix applies to both market price and the static production price.

In fact, the market price system should satisfy

(12) $$\bar{p}^t = \bar{p}^t \cdot M_t(I + \hat{r}),$$

and the market equilibrium price

(13) $$\bar{p}^t = (1+r)\bar{p}^t \cdot M_t.$$

Thus, in a Leontief economy, goods are exchanged at their static production price through the atomistic market behaviour of capitalists.

§ 3. Technical changes and von Neumann economies.

1. A very nice result was obtained at the end of the preceding section in the Leontief economy case. Our next task is to inquire the following—is it possible to extend that result to von Neumann economies with (or without) technical changes?

Let us first consider a von Neumann economy without technical changes. The market price of this case may be written as

(13) $$\bar{p}^t \cdot B\hat{x}^t = \bar{p}^t(M\hat{x}^t + U_t),$$

which can be reduced to

(14) $$\bar{p}^t B = \bar{p}^t \cdot M(I + \hat{r}).$$

If we apply the concepts introduced in § 1, it will be seen that the market equilibrium price determined by (9) as well as (14) is one of the feasible solutions for (I-24). It may be, however, an optimum solution for (I-24). Hence, there is only a slight possibility for the market to find the static von Neumann production price.

2. If we consider a von Neumann economy with technical changes, it is immediately seen that the possibility is lost.

In fact, supply of goods is expressed by $\bar{p}^t \cdot B_t \hat{x}^t$, whereas demand of goods by $\bar{p}^t(M_{t+1}\hat{x}^{t+1}+U_t)$. Hence, the market price system is expresed by

(15) $$\bar{p}^t \cdot B_t = \bar{p}^t \cdot M_{t+1}(I+\hat{r}),$$

and the market equilibrium price is the one to satisfy (9).

Now, it is evident that except such a specific case as discussed in Proposition 1. 3, the market equilibrium price is not equal to the static production price in general. The production price is one thing, the market equilibrium price is another.

EPILOGUE
Tentative Conclusions

1. By way of conclusions, we try to mention some of the implications derived from our investigation.

2. The first overall point is that if we date things and consider the dynamic production system, it becomes clear that production price equilbria are not free from quantities even if techniques are linear: production price equilibrium depend on a certain state of economies.

3. Now, we have seen so far that the development of the reproduction process alternates the transformation of inputs into outputs, i.e., production, and that of outputs into input, i.e., exchange. Depending on two types of transformation of goods, two types of prices should be distinguished. The one is the production price associated with the transformation of inputs into outputs. The other is the market price associated with exchange.

The production price is originally established as dynamic one linking goods with different dates, and the dynamic production price essentially converges to the static production price. The profit rate is grasped as the time factor concerning different dates. It is worth noting that the production price supports the accumulation of capital as discussed by the turnpike theory.

On the other hand, the market price is established by equating supply and demand which are synchronised, so that it is timeless.

Then, one of the remarkable features of the Leontief economy is that even if there is a technical change in the economy, the market equilibrium price can be equal to the static production price. That is, if all the agents behave in the market to maximise their individual interests by harmonizing their growth rates, then the market equilibrium price equals the static production price which guarantees the optimum accumulation of capital. The market supports the accumulation of capital.

If we take joint-production into account, however, we see that the above nice feature is lost.

Let all the agents behave independently to pursue their own interests, and the market equilibrium price will be established which does not support the optimum accumulation of capital. It must be stressed that even if the market is complete and the supply-demand adjustment is smooth and stable, there is a discrepancy between the two price equilibria.

Depending on the macroscopic development of reproduction that is based on the production of commodities by means of commmodities, there appear two major centres of gravity in the economy. Thus, we can say that the market economy is one thing, the accumulation of capital is another.

4. Our investigation may shed light on the Marxian theory of business cycles.

One of its points is that through one business cycle fluctuations in the market price are offset to establish the production price as the average. This is known as the average mechanism of business cycles. It has not yet been made clear, however, why such an average mechanism ought to involve a catastrophe or crisis. Our investigation may suggest that crises are inevitable because the average mechanism must bring about what is essentially different from the market equilibrium price. A business cycle may be the process in which production and exchange, or the accumulation of capital and the market, conflict with each other.

5. We have pointed out that the actual economic process possesses two conflicting centres of movement. Now, it is not so difficult to understand why two different types of price theory have been developed. Different schools of economics have placed stress on different centres of movement, or the phases of the reproduction process. This implies that the neoclassical price theory which mostly discusses exchange is not sufficient as the price theory, and, at the same time, that the alternative price theory which should not only deal with production but also treat the synthesis of production and exchange is necessary. Any way, there will be no dougt about the necessity to reconsider the price theory on the macroscopic viewpoint.

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