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A Study of the Evolution of Stock Price Indices in Terms of Oscillation Theory

Part I. Statistical Properties

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Abstract

Considering that stock price indices oscillate in their evolution, this series of papers proposes to analyze the fluctuation of stock price indices in light of a theory of oscillation. The time evolution of the Nikkei 225, for example, is analyzed and some statistical properties such as the frequency distribution of shifts of phases are discussed in terms of oscillation theory. The results are also compared with those from a standard random walk theory.

Introduction

A new research field called "Econophysics" has been born very recently[1], which stands for "Economics" + "Physics". In econophysics one studies various phenomena in finances and securities in terms of physics tools as well as mathematics. It seems that phenomena in securities, for example, are too complex to handle in a similar way as done in natural sciences. The idea of econophysics is to find a simple law or dynamics behind its complicated phenomenon by reducing the system to be as simple as possible with introducing several assumptions.

In the present paper we study the time evolution of stock prices in terms of physics tools. The number of stock issues is extremely large. For example, 1,600 companies are the member of Tokyo Stock Market. Therefore we restrict ourselves to study a specific index such as Nikkei 225 instead of dealing with each issue. It appears that stock prices vary randomly without any causal laws. It seems that a stock price in this month varies independently of the price in last month. Nevertheless, we introduce a theory for oscillations in order to analyze the time evolution of the stock prices[2-3]. The theory of oscillations is a very common approach in physics, which has an advantage to be solved analytically and to predict future development.

The present paper is organized as follows. In section 1 we present an analytic formulation for the time evolution of the stock price indices in terms of the theory for oscillations. In section 2 we also give an analytic expression derived from a standard random walk theory. In section 3 we apply these formalisms to Nikkei 225 data of Tokyo Stock Market. We compare results derived by the present theory with those by the random walk theory.

1. Analysis in Terms of Theory for Oscillations

In this section, we present the analysis of stock price fluctuation in terms of oscillation theory and the classification of the fluctuation modes.

1.1 *Oscillatory Fluctuations*

Let us assume that stock prices fluctuate according to the theory for oscillations. Namely a stock price index $x(t)$ (for example, Nikkei 225) at a time t oscillates around a mean value b. Then $x(t)$ is determined by the following differential equation,

$$\frac{d^2}{dt^2}x(t) = -\omega^2\{x(t)-b\}, \tag{1.1}$$

where ω is the angular frequency, which represents the speed of oscillations. It is quite standard to solve equation (1.1). The solution is given as follows,

$$x(t) = a\cos(\omega t+\theta)+b, \tag{1.2}$$

where a (amplitude) and θ (phase) are parameters in this formalism. In the present case, we treat ω and b as parameters as well. If four initial values are given for x at various time t, the parameters are determined uniquely. Therefore the stock price indices $x(t)$ at later time is determined by equation (1.2).

Let $\varDelta t$ be a constant time interval. Then the indices at $t = 0$, $\varDelta t$, $2\varDelta t$, $3\varDelta t$ are presented as follows,

$$x(0) = a\cos(\theta)+b, \tag{1.3}$$
$$x(\varDelta t) = a\cos(\omega\varDelta t+\theta)+b, \tag{1.4}$$
$$x(2\varDelta t) = a\cos(2\omega\varDelta t+\theta)+b, \tag{1.5}$$
$$x(3\varDelta t) = a\cos(3\omega\varDelta t+\theta)+b. \tag{1.6}$$

The variation of x between two successive time points is defined as follows,

$$\begin{aligned} y\left(\frac{1}{2}\varDelta t\right) &= x(\varDelta t)-x(0) = a\cos(\omega\varDelta t+\theta)-a\cos(\theta) \\ &= -2a\sin\left(\frac{1}{2}\omega\varDelta t\right)\sin\left(\frac{1}{2}\omega\varDelta t+\theta\right), \end{aligned} \tag{1.7}$$

$$\begin{aligned} y\left(\frac{3}{2}\varDelta t\right) &= x(2\varDelta t)-x(\varDelta t) = a\cos(2\omega\varDelta t+\theta)-a\cos(\omega\varDelta t+\theta) \\ &= -2a\sin\left(\frac{1}{2}\omega\varDelta t\right)\sin\left(\frac{3}{2}\omega\varDelta t+\theta\right), \end{aligned} \tag{1.8}$$

$$\begin{aligned} y\left(\frac{5}{2}\varDelta t\right) &= x(3\varDelta t)-x(2\varDelta t) = a\cos(3\omega\varDelta t+\theta)-a\cos(2\omega\varDelta t+\theta) \\ &= -2a\sin\left(\frac{1}{2}\omega\varDelta t\right)\sin\left(\frac{5}{2}\omega\varDelta t+\theta\right). \end{aligned} \tag{1.9}$$

We also define the variation of y between two successive time points as follows,

$$z(\Delta t) = y\left(\frac{3}{2}\Delta t\right) - y\left(\frac{1}{2}\Delta t\right) = -4a\sin^2\left(\frac{1}{2}\omega\Delta t\right)\cos(\omega\Delta t + \theta), \tag{1.10}$$

$$z(2\Delta t) = y\left(\frac{5}{2}\Delta t\right) - y\left(\frac{3}{2}\Delta t\right) = -4a\sin^2\left(\frac{1}{2}\omega\Delta t\right)\cos(2\omega\Delta t + \theta). \tag{1.11}$$

Similarly, the variation of z between two successive time points is defined by

$$w\left(\frac{3}{2}\Delta t\right) = z(2\Delta t) - z(\Delta t) = 8a\sin^3\left(\frac{1}{2}\omega\Delta t\right)\sin\left(\frac{3}{2}\omega\Delta t + \theta\right). \tag{1.12}$$

Using equations (1.8) and (1.12), we find the ratio w/y at a time $\frac{3}{2}\Delta t$ as follows,

$$R_{w/y} \equiv \frac{w\left(\frac{3}{2}\Delta t\right)}{y\left(\frac{3}{2}\Delta t\right)} = -4\sin^2\left(\frac{1}{2}\omega\Delta t\right). \tag{1.13}$$

Equation (1.13) gives an important condition for oscillatory fluctuations. That is

$$-4 < R_{w/y} < 0 \tag{1.14}$$

for any oscillatory fluctuations.

Finally, one can predict a stock price index at $t = 4\Delta t$ in terms of the previous values at $t = 0$, Δt, $2\Delta t$, $3\Delta t$ as follows,

$$x(4\Delta t) = x(\Delta t) + \frac{(x(3\Delta t) - x(2\Delta t))(x(3\Delta t) - x(0))}{x(2\Delta t) - x(\Delta t)}. \tag{1.15}$$

1.2 *Zigzag Fluctuations*

In some cases, the stock price indices do not satisfy the condition of equation (1.14), but the following condition,

$$R_{w/y} < -4. \tag{1.16}$$

In this case, no analytical solutions of the differential equation (1.1) are available. Therefore fluctuation is not oscillatory. Instead the fluctuation of the indices is classified into following two cases.

(i) The case $w\left(\frac{3}{2}\Delta t\right) > 0$, when we can show that

$$x(2\Delta t) < x(\Delta t),\ x(3\Delta t) > x(0). \tag{1.17}$$

In this case the stock price index shows a tendency to rise in zigzag motions. We call this fluctuation mode as rising tendency.

(ii) The case $w\left(\frac{3}{2}\Delta t\right) < 0$, when we can show that

$$x(2\Delta t) > x(\Delta t),\ x(3\Delta t) < x(0). \tag{1.18}$$

In this case the stock price index shows a tendency to decrease in zigzag motions. We call this fluctuation mode as decreasing tendency.

In the case of the zigzag fluctuations, one can no longer express $x(t)$ in analytic form. If the following condition

$$\frac{w\left(\frac{5}{2}\Delta t\right)}{y\left(\frac{5}{2}\Delta t\right)} = \frac{w\left(\frac{3}{2}\Delta t\right)}{y\left(\frac{3}{2}\Delta t\right)} \tag{1.19}$$

is satisfied, however, one can derive the relation of $x(t)$ in the same form as equation (1.15). Namely,

$$x(4\Delta t) = x(\Delta t) + \frac{(x(3\Delta t) - x(2\Delta t))(x(3\Delta t) - x(0))}{x(2\Delta t) - x(\Delta t)}, \tag{1.20}$$

which shows that the index at $t = 4\Delta t$ is predicted again.

1.3 *Divergent Fluctuations and Unstable Fluctuations*

In the case of oscillatory fluctuations the force acting on the stock price indices was attractive as in equation (1.1), which stabilizes the system. If the force is repulsive, on the other hand, the system becomes unstable. In this case, the differential equation for the index fluctuation is

$$\frac{d^2}{dt^2} x(t) = \omega^2 \{x(t) - b\}. \tag{1.21}$$

The solution is given as

$$x(t) = a \cosh(\omega t + \theta) + b, \tag{1.22}$$

where cosh is the hyperbolic cosine function. The parameters a, θ, ω and b appear similarly to those defined in section 1.1. One can repeat the same argument as in section 1.1. Stock price indices at time $t = 0$, Δt, $2\Delta t$, $3\Delta t$ are expressed as follows.

$$x(0) = a \cosh(\theta) + b, \tag{1.23}$$

$$x(\Delta t) = a \cosh(\omega \Delta t + \theta) + b, \tag{1.24}$$

$$x(2\Delta t) = a \cosh(2\omega \Delta t + \theta) + b, \tag{1.25}$$

$$x(3\Delta t) = a \cosh(3\omega \Delta t + \theta) + b. \tag{1.26}$$

The variation of x between two successive time points is defined as follows,

$$y\left(\frac{1}{2}\Delta t\right) = x(\Delta t) - x(0) = 2a \sinh\left(\frac{1}{2}\omega \Delta t\right) \sinh\left(\frac{1}{2}\omega \Delta t + \theta\right), \tag{1.27}$$

$$y\left(\frac{3}{2}\Delta t\right) = x(2\Delta t) - x(\Delta t) = 2a \sinh\left(\frac{1}{2}\omega \Delta t\right) \sinh\left(\frac{3}{2}\omega \Delta t + \theta\right), \tag{1.28}$$

$$y\left(\frac{5}{2}\Delta t\right) = x(3\Delta t) - x(2\Delta t) = 2a \sinh\left(\frac{1}{2}\omega \Delta t\right) \sinh\left(\frac{5}{2}\omega \Delta t + \theta\right). \tag{1.29}$$

We also define the variation of y between two successive time points as follows,

$$z(\Delta t) = y\left(\frac{3}{2}\Delta t\right) - y\left(\frac{1}{2}\Delta t\right) = 4a \sinh^2\left(\frac{1}{2}\omega \Delta t\right) \cosh(\omega \Delta t + \theta), \tag{1.30}$$

$$z(2\Delta t) = y\left(\frac{5}{2}\Delta t\right) - y\left(\frac{3}{2}\Delta t\right) = 4a \sinh^2\left(\frac{1}{2}\omega\Delta t\right)\cosh(2\omega\Delta t+\theta). \quad (1.31)$$

Similarly, the variation of z between two successive time points is defined by

$$w\left(\frac{3}{2}\Delta t\right) = z(2\Delta t) - z(\Delta t) = 8a \sinh^3\left(\frac{1}{2}\omega\Delta t\right)\sinh\left(\frac{3}{2}\omega\Delta t+\theta\right). \quad (1.32)$$

Using equations (1.28) and (1.32), we find the ratio w/y at a time $\frac{3}{2}\Delta t$ as follows,

$$R_{w/y} \equiv \frac{w\left(\frac{3}{2}\Delta t\right)}{y\left(\frac{3}{2}\Delta t\right)} = 4 \sinh^2\left(\frac{1}{2}\omega\Delta t\right). \quad (1.33)$$

Equation (1.33) gives an important condition for the present fluctuation mode. Namely,

$$R_{w/y} > 0. \quad (1.34)$$

If the stock price index satisfies equation (1.34), the fluctuation becomes either divergent or unstable. One can classify the fluctuation into the following four cases.

(i) The case $z(2\Delta t) > 0$ and $w\left(\frac{3}{2}\Delta t\right) > 0$, when we can show that

$$y\left(\frac{3}{2}\Delta t\right) > 0,\ y\left(\frac{5}{2}\Delta t\right) > y\left(\frac{3}{2}\Delta t\right) > y\left(\frac{1}{2}\Delta t\right),\ z(2\Delta t) > z(\Delta t) > 0. \quad (1.35)$$

In this case the stock price index increases divergently. We call this fluctuation mode divergent rise.

(ii) The case $z(2\Delta t) < 0$ and $w\left(\frac{3}{2}\Delta t\right) < 0$, when we can show that

$$y\left(\frac{3}{2}\Delta t\right) < 0,\ y\left(\frac{5}{2}\Delta t\right) < y\left(\frac{3}{2}\Delta t\right) < y\left(\frac{1}{2}\Delta t\right),\ z(2\Delta t) < z(\Delta t) < 0. \quad (1.36)$$

In this case the stock price index decreases divergently. We call this fluctuation mode divergent decrease.

(iii) The case $z(2\Delta t) > 0$ and $w\left(\frac{3}{2}\Delta t\right) < 0$, when we can show that

$$y\left(\frac{1}{2}\Delta t\right) < y\left(\frac{3}{2}\Delta t\right) < 0,\ y\left(\frac{3}{2}\Delta t\right) < y\left(\frac{5}{2}\Delta t\right),\ z(2\Delta t) < z(\Delta t),\ z(\Delta t) > 0. \quad (1.37)$$

In this case the stock price index decreases step by step. For each step the amplitude of fluctuations approaches to zero, therefore it is unpredictable whether the index increases or decreases in further steps. We call it unstable dumping fluctuation.

(iv) The case $z(2\Delta t) < 0$ and $w\left(\frac{3}{2}\Delta t\right) > 0$, when we can show that

$$y\left(\frac{1}{2}\Delta t\right) > y\left(\frac{3}{2}\Delta t\right) > 0,\ y\left(\frac{3}{2}\Delta t\right) > y\left(\frac{5}{2}\Delta t\right),\ z(2\Delta t) > z(\Delta t),\ z(\Delta t) < 0. \quad (1.38)$$

In this case the stock price index increases step by step. For each step the amplitude of fluctuations approaches to zero, therefore it is also unpredictable whether the index increases or decreases in further steps. We again call it unstable dumping fluctuation.

In these four cases, one can again predict a value at $t=4\Delta t$ in terms of the previous four values at $t=0$, Δt, $2\Delta t$, $3\Delta t$ as follows,

$$x(4\Delta t)=x(\Delta t)+\frac{(x(3\Delta t)-x(2\Delta t))(x(3\Delta t)-x(0))}{x(2\Delta t)-x(\Delta t)}. \tag{1.39}$$

2. Analysis in Theory for Random Walks

Let us define $x(t)$ as a stock price index at a given time t and assume that the index follows the theory of random walks[4]. We use the same definitions for x, y, z and w as given in section 1. (Readers should refer to equations (1.7)-(1.13) for their explicit definitions.) The definition of the theory of random walks is as follows. We assume that the probability distribution function for y is given by the normal distribution $N(0, \sigma^2)$, where the average of the distribution is zero and σ is the standard deviation. The explicit form of the probability distribution function is given by

$$\rho(y)=\frac{1}{\sqrt{2\pi\sigma}}\exp\left(-\frac{y^2}{2\sigma^2}\right). \tag{2.1}$$

Similarly, the probability distribution function for finding the three values of (y_1, y_2, y_3) for $y_1=y\left(\frac{1}{2}\Delta t\right)$, $y_2=y\left(\frac{3}{2}\Delta t\right)$, and $y_3=y\left(\frac{5}{2}\Delta t\right)$ is given as follows,

$$\rho(y_1, y_2, y_3)=\left(\frac{1}{\sqrt{2\pi\sigma}}\right)^3\exp\left\{-\frac{1}{2\sigma^2}(y_1^2+y_2^2+y_3^2)\right\}. \tag{2.2}$$

The total probability is normalized to unity as shown by the following expression,

$$\int_{-\infty}^{+\infty}dy_1\int_{-\infty}^{+\infty}dy_2\int_{-\infty}^{+\infty}dy_3\rho(y_1, y_2, y_3)=1. \tag{2.3}$$

In the analysis of the stock price indices in terms of the theory for oscillations, the ratio $R_{w/y}\equiv w/y$ played an essential role in classifying the fluctuations as shown in last section. Therefore we formulate the expression for $R_{w/y}$ in the theory of random walks. The definition of $R_{w/y}$ is given by

$$R_{w/y}\equiv\frac{w\left(\frac{3}{2}\Delta t\right)}{y\left(\frac{3}{2}\Delta t\right)}. \tag{2.4}$$

Let us define $P_{\Delta s}(s)$ as a probability for finding $R_{w/y}$ in the following interval,

$$s\leq R_{w/y}\leq s+\Delta s, \tag{2.5}$$

where $0<\Delta s\ll 1$. One can express $P_{\Delta s}(s)$ as follows,

$$
\begin{aligned}
P_{\Delta s}(s)\Delta s = &\int_{-\infty}^{+\infty} dy_1 \int_{0}^{+\infty} dy_2 \int_{-y_1+(s+2)y_2}^{-y_1+(s+\Delta s+2)y_2} dy_3 \rho(y_1, y_2, y_3) \\
&+ \int_{-\infty}^{+\infty} dy_1 \int_{-\infty}^{0} dy_2 \int_{-y_1+(s+\Delta s+2)y_2}^{-y_1+(s+2)y_2} dy_3 \rho(y_1, y_2, y_3).
\end{aligned}
\tag{2.6}
$$

Inserting equation (2.2) into equation (2.6) one can perform three dimensional integration in equation (2.6). After lengthy but straightforward calculations, one obtains the following expression for $P_{\Delta s}(s)$.

$$
P_{\Delta s}(s) = \frac{1}{\Delta s}\frac{1}{\pi}\left\{\sin^{-1}\left(\frac{s+\Delta s+2}{\sqrt{(s+\Delta s+2)^2+2}}\right) - \sin^{-1}\left(\frac{s+2}{\sqrt{(s+2)^2+2}}\right)\right\}. \tag{2.7}
$$

By taking a limit of $\Delta s \rightarrow 0$, equation (2.7) is further simplified as

$$
P(s) \equiv \lim_{\Delta s \rightarrow 0} P_{\Delta s}(s) = \frac{\sqrt{2}}{\pi}\frac{1}{(s+2)^2+2}, \tag{2.8}
$$

where $P(s)\,ds$ denotes the probability for finding $R_{w/y}$ in the interval between s and $s+ds$. We note that $P(s)$ is normalized to unity as

$$
\int_{-\infty}^{+\infty} ds P(s) = 1. \tag{2.9}
$$

Equation (2.8) is an extremely important result of the random walk theory. We summarize the predictions derived from equation (2.8) as follows,

(i) For oscillatory fluctuations: $-4 < R_{w/y} < 0$, we obtain the probability

$$
P_{\mathrm{oscill}} \equiv \int_{-4}^{0} ds P(s) = \frac{2}{\pi}\sin^{-1}\left(\sqrt{\frac{2}{3}}\right) = 0.608. \tag{2.10}
$$

(ii) For zigzag fluctuations: $R_{w/y} < -4$, we obtain the probability

$$
P_{\mathrm{zigzag}} \equiv \int_{-\infty}^{-4} ds P(s) = \frac{1}{2} - \frac{1}{\pi}\sin^{-1}\left(\sqrt{\frac{2}{3}}\right) = 0.196. \tag{2.11}
$$

The probability of zigzag fluctuations is furthermore divided into two parts as

$$
P_{\mathrm{zigzag}}(w > 0) = P_{\mathrm{zigzag}}(w < 0) = \frac{1}{2}P_{\mathrm{zigzag}} = 0.098. \tag{2.12}
$$

(iii) For divergent and unstable fluctuations: $R_{w/y} > 0$, we obtain the probability

$$
P_{\mathrm{div}} \equiv \int_{0}^{+\infty} ds P(s) = \frac{1}{2} - \frac{1}{\pi}\sin^{-1}\left(\sqrt{\frac{2}{3}}\right) = 0.196. \tag{2.13}
$$

The probability for divergent and unstable fluctuations is furthermore divided into four parts as

$$
P_{\mathrm{div}}(z > 0,\ w > 0) = P_{\mathrm{div}}(z < 0,\ w < 0) = \frac{7}{48} - \frac{1}{4\pi}\sin^{-1}\left(\sqrt{\frac{2}{3}}\right) = 0.070, \tag{2.14}
$$

$$
P_{\mathrm{div}}(z > 0,\ w < 0) = P_{\mathrm{div}}(z < 0,\ w > 0) = \frac{5}{48} - \frac{1}{4\pi}\sin^{-1}\left(\sqrt{\frac{2}{3}}\right) = 0.028. \tag{2.15}
$$

In the next section we will compare these predictions of the random walk theory with a practical stock price index Nikkei 225 of Tokyo Stock Market[5].

3. Application to Nikkei 225 data of Tokyo Stock Market

In this section we analyze stock price data in terms of the theory for oscillations derived in section 1. In the present paper, in particular, we study general trends of stock prices. Therefore, we use Nikkei 225 data instead of individual issues. In order to reduce statistical uncertainties as much as possible, we try to include large number of data available for us.

3.1 *Frequency Distributions for Fluctuation Modes*

In Figure 1 we have plotted a histogram of Nikkei 225 data as a function of $R_{w/y}$. The data are Nikkei 225 stock price indices at the end of months. The total number of data is 400 (months) for the last 33 years in the period of 1970–2003. Each bar in Figure 1 denotes the number of frequency for the interval $\varDelta R_{w/y} = 0.1$. The solid curve is the prediction of the random walk theory given by equation (2.8), which is normalized to the total number of frequencies. As far as the gross structure of the frequency distribution is concerned, both of the shape and the peak position of the frequency distribution of the practical data seem to be consistent with the random walk theory prediction. However, the number of data points (400) is not sufficient enough to exclude statistical errors.

In Table 1 we have calculated the frequencies (in percent) of the same Nikkei 225 data for the period of 1970–2003 for each fluctuation mode. The first, second and third lines stand for oscillation fluctuations, zigzag fluctuations and divergent fluctuations, respectively. The second column shows the prediction by the random walk theory given by equations (2.10)–(2.15). Again, the gross structure of the frequency distribution of Nikkei 225 seems to be

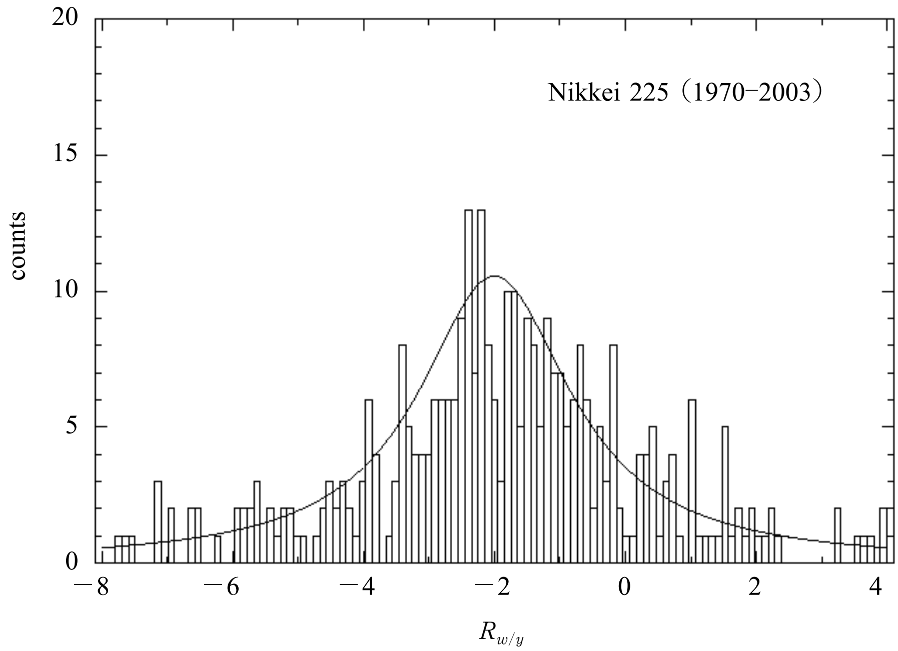

Figure 1　Frequency distribution of Nikkei 225 data for the last 33 years as a function of $R_{w/y}$.
The total number of data is 400 (months) for the period of 1970–2003. The curve is a result of the random walk theory (equation (2.8)), which is normalized to the total number of data.

Table 1 Frequency distribution of Nikkei 225 data for the last 33 years in various fluctuation modes. The total number of data is 400 (months) for the period of 1970–2003.

Fluctuations		Nikkei 225 Data		Random Walk Theory	
Oscillations		0.595		0.608	
Zigzag	(i) $w>0$	0.1225	0.215	0.098	0.196
	(ii) $w<0$	0.0925		0.098	
Divergent	(i) $z>0,\ w>0$	0.0800	0.190	0.070	0.196
	(ii) $z<0,\ w<0$	0.0525		0.070	
	(iii) $z>0,\ w<0$	0.0225		0.028	
	(iv) $z<0,\ w>0$	0.0350		0.028	
Total		1.000		1.000	

described by the random walk theory reasonably well. However, there exists a significant deviation in specific fluctuation modes. Again higher statistics is necessary to draw a definite conclusion.

3.2 *Phases in Time Evolutions*

In order to analyze the time evolutions of practical data of stock price indices we introduce an idea of "phase". It is commonly understood in physics that phase and amplitude describe oscillations completely. In Figure 2 we show a diagram of a typical periodic oscillation, and we have assigned the phases I–IV. In the case of perfect periodic oscillations, the four phases will be repeated in a successive order from I to IV. We define the phases as follows, phase I: $y>0$, $z<0$, phase II: $y<0$, $z<0$, phase III: $y<0$, $z>0$, and phase IV: $y>0$, $z>0$.

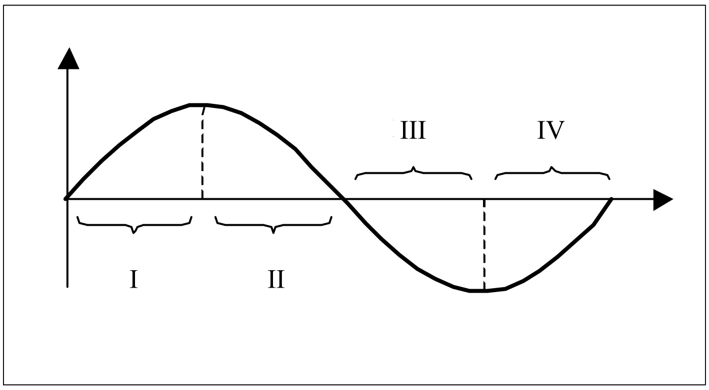

Figure 2 Definition of phases in a periodic oscillation.

In Table 2 we have shown the frequency for shifts of the phase among the four phases for Nikkei 225 data for the period of 1970–2003 and have compared with the results from random walk theory calculation. The agreement is surprisingly well, although the number of data points (400) is not large enough to draw a definite conclusion. It is noticeable, however, that Nikkei 225 changes the phase from I to II and from III to IV in order more frequently than the random walk theory prediction. These shifts of the phase take place, when the indices change the direction of fluctuation. The present notice may indicate that practical indices fluctuate remembering its history.

Table 2 Frequency of shifts among the four phases for Nikkei 225 data and for random walk theory calculations.

The total number of data for Nikkei 225 is 400 (months) for the period of 1970–2003.

(1) Nikkei 225 data (1970–2003)

previous month \ next month	I	II	III	IV
I	19%	50%	20%	11%
II	7%	19%	55%	19%
III	17%	2%	10%	71%
IV	58%	13%	10%	19%

(2) Random Walk Theory

previous month \ next month	I	II	III	IV
I	19%	42%	32%	6%
II	4%	20%	64%	12%
III	23%	6%	20%	51%
IV	59%	14%	17%	10%

In summary we have introduced a formulation for the evolution of the stock prices indices in terms of a theory for oscillations. We have shown that the ratio of w/y is an essential quantity in analyzing the stock price indices. We have compared the data with a standard random walk theory result. Several general features of the Nikkei 225 data for the last 33 years such as frequency distributions for fluctuation modes and for shifts of phases are well described by the random walk theory. The frequency distributions for shifts of the phase may, however, indicate that practical indices fluctuate remembering its history.

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