Fuzzy Multi-Criteria Minimum Spanning Tree Problem

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Abstract

In this paper, a fuzzy multi-criteria minimum spanning tree problem is formulated as expected minimum spanning tree model, α-minimum spanning tree model and the most minimum spanning tree model according to different decision criteria. Then the crisp equivalents are derived when the fuzzy costs are characterized by triangular fuzzy numbers. Furthermore, a simulation-based genetic algorithm using Prüfer number representation is designed. Finally, a numerical example is given to illustrate the effectiveness of the algorithm.

Keywords: Minimum spanning tree, fuzzy programming, genetic algorithm

1 Introduction

The Minimum Spanning Tree (MST) problem is to find a spanning tree with the minimal weight in an edge weighted graph. It is a classical network optimization problem, and has important applications in transportation, communications, distribution systems, etc.

On the background of Boruvka's pioneer work, the MST problem has been well studied. Kruskal [9], Prim [18] and Gabow [5] made great contribution to the efficient algorithm. These works made the MST problem popular and have further development.

Generalized from a single objective problem, the multi-criteria MST problem was introduced when there are multiple attributes defined on each edge. The problem may arise, for instance, when designing a layout for telecommunication system, besides the cost for connection between cities or terminals, other factors such as the time for communication or construction, the complexity for construction and even the reliability are all important and have to be taken into consideration. In all these cases, the MST with multi-criteria is a very realistic representation of the practical problem.

In real-life conditions, we know some information with uncertainty. For example, the parameters of problem are vague or in a subjective nature. Then the parameters may be specialized as fuzzy variables by an experts system. In [8], Itoh and Ishii formulated an MST problem with fuzzy cost as chance-constrained programming based on the necessity measure. In [2], Chang and Lee defined three means based on the Overall Existence Ranking Index [1] for ranking fuzzy costs of spanning trees. Recently, Liu [17] developed a credibility theory including credibility measure, critical value and expected value to rank fuzzy variables. In this paper, based on the credibility theory, we propose the concepts of expected minimum spanning tree (EMST), α-minimum spanning tree (α-MST) and the most minimum spanning tree (MMST) in a fuzzy multi-criteria minimum spanning tree (FMCMST) problem. Then in order to find the EMST, α-MST and MMST, we formulate the FMCMST problem as expected minimum spanning tree model, α-minimum spanning tree model and the most minimum spanning tree model, respectively. Then crisp equivalents are discussed when fuzzy weights are characterized by triangular fuzzy numbers.

This paper is arranged as follows. After recalling some preliminaries in credibility theory, Section 3 introduces the concepts of EMST, α-MST and MMST. Section 4 proposes three types of fuzzy programming models for an FMCMST problem. Then the crisp equivalents of the three models are discussed in Section 5. Section 6 discusses in detail genetic algorithm for the FMCMST problem. In Section 7, a numerical example is given to show the effectiveness of the algorithm and the conclusion follows in Section 8.

2 Preliminaries

Possibility Theory was proposed by Zadeh [19], and developed by many researchers such as Dubois and Prade [4]. Let Θ be a nonempty set, $\mathcal{P}(\Theta)$ the power set of Θ , and Pos a possibility measure. The triplet $(\Theta, \mathcal{P}(\Theta), \text{Pos})$ is called a possibility space. Let ξ be a fuzzy variable with membership function μ, then for any set B of real numbers, we have $\text{Pos}\{\xi \in B\} = \sup_{x\in B}\mu(x)$ and $\text{Nec}\{\xi \in B\} = 1 - \sup_{x\in B^c}\mu(x)$. It is obvious that a fuzzy event may fail even though its possibility achieves 1, and hold even though its necessity is 0.

In order to rank fuzzy variables, we introduce the credibility theory developed by Liu [17], which includes credibility measure, expected value and critical value to rank fuzzy variables.

The credibility measure Cr is defined by Liu and Liu [15] as the average of possibility measure and necessity measure, i.e.,

$$\text{Cr}\{\xi \le r\} = \frac{1}{2}(\text{Pos}\{\xi \le r\} + \text{Nec}\{\xi \le r\}). \tag{1}$$

Based on the credibility measure, we have the expected value operator as follows.

Definition 1 (Liu and Liu [15]) *Let ξ be a fuzzy variable. The expected value of ξ is defined as*

$$\text{E}[\xi] = \int_0^{\infty} \text{Cr}\{\xi \ge r\}\text{d}r - \int_{-\infty}^{0} \text{Cr}\{\xi \le r\}\text{d}r, \tag{2}$$

provided that both integrals are finite.

Let ξ and η be independent fuzzy variables. Then for any real numbers a and b, we have $\text{E}[a\xi + b\eta] = a\text{E}[\xi] + b\text{E}[\eta]$.

Definition 2 *Let ξ be a fuzzy variable, and $\alpha \in (0, 1]$. Then*

$$\xi_{\sup}(\alpha) = \sup\{r \mid \text{Cr}\{\xi \ge r\} \ge \alpha\} \tag{3}$$

is called the α-optimistic value to ξ; and

$$\xi_{\inf}(\alpha) = \inf\{r \mid \text{Cr}\{\xi \le r\} \ge \alpha\} \tag{4}$$

is called the α-pessimistic value to ξ.

3 Fuzzy Multi-Criteria Minimum Spanning Tree

Let $G = (V, E)$ be an undirected graph with vertex set $V = \{v_1, v_2, \cdots, v_n\}$ and edge set $E = \{e_1, e_2, \cdots, e_m\}$. A spanning tree $T = (V, S)$ is a subgraph of G such that $S \subseteq E$, $|S| = n - 1$ (where $|S|$ denotes the cardinality of set S) and T is connected. Each edge has p associated fuzzy variables, representing p attributes defined on it and denoted with $\boldsymbol{\xi}_i = (\xi_{1i}, \xi_{2i}, \cdots, \xi_{pi})(i = 1, 2, \cdots, m)$. In practice $\xi_{li}(l = 1, 2, \cdots, p)$ may represent the distance, cost, and so on. Let $\boldsymbol{x}$ be a binary decision variable defined as:

$$x_i = \begin{cases} 1, & \text{if edge } e_i \text{ is selected} \\ 0, & \text{otherwise.} \end{cases} \tag{5}$$

Then a spanning tree of graph G can be expressed by the vector $\boldsymbol{x}$. Let X be the set of all such vectors corresponding to spanning trees in graph G, the FMMST problem can be formulated as:

$$\begin{cases} \min \left\{ z_1(\boldsymbol{x},\boldsymbol{\xi}) = \sum\limits_{i=1}^{m} \xi_{1i}x_i, z_2(\boldsymbol{x},\boldsymbol{\xi}) = \sum\limits_{i=1}^{m} \xi_{2i}x_i, \cdots, z_p(\boldsymbol{x},\boldsymbol{\xi}) = \sum\limits_{i=1}^{m} \xi_{pi}x_i \right\} \\ \text{subject to:} \\ \qquad x_i = 0 \text{ or } 1 \\ \qquad \boldsymbol{x} \in X. \end{cases} \tag{6}$$

If the decision maker has a real-valued preference function aggregating the p objective functions, then we may minimize the aggregating preference function subject to the same set of constraints. This model is referred to as a compromise model whose solution is called a compromise solution. The following model is one of such compromise models.

$$\begin{cases} \min z(\boldsymbol{x},\boldsymbol{\xi}) = \sum\limits_{l=1}^{p} \lambda_l z_l(\boldsymbol{x},\boldsymbol{\xi}) \\ \text{subject to:} \\ \qquad \boldsymbol{x} \in X, \end{cases} \tag{7}$$

where the weights $\lambda_1, \lambda_2, \cdots, \lambda_p$ are nonnegative numbers with $\lambda_1 + \lambda_2 + \cdots + \lambda_p = 1$. Note that the solution of (7) must be a Pareto solution of the original one.

In real-life conditions, the decision maker often faces with some insufficient information about the weights. For these cases, the distance, cost and other attributes $\boldsymbol{\xi}_i = (\xi_{1i}, \xi_{2i}, \cdots, \xi_{pi})$ $(i = 1, 2, \cdots, m)$ may be specified as fuzzy variables according to the expert system. Then the objective functions $z_l(\boldsymbol{x}), (l = 1, 2, \cdots, p)$ become fuzzy variables, too. In order to rank spanning trees with fuzzy weights, different decision makers may have different ideas. Suppose that the decision maker hopes to minimize the expected value of the fuzzy weights, we present the concept of expected minimum spanning tree (EMST).

Definition 3 *A spanning tree $\boldsymbol{x}^*$ is called the expected minimum spanning tree if*

$$\mathrm{E}[z(\boldsymbol{x}^*,\boldsymbol{\xi})] \le \mathrm{E}[z(\boldsymbol{x},\boldsymbol{\xi})]$$

for all spanning tree $\boldsymbol{x} \in X$, where $\mathrm{E}[z(\boldsymbol{x}^*,\boldsymbol{\xi})]$ is called the expected minimum cost.

In many cases, the decision maker sets a confidence level α as an appropriate safety margin, and hopes to minimize a critical value $\bar{z}$ with $\mathrm{Cr}\{z(\boldsymbol{x},\boldsymbol{\xi}) \le \bar{z}\} \ge \alpha$. For this case, we propose the concept of α-minimum spanning tree (α-MST) as follows:

Definition 4 *A spanning tree $\boldsymbol{x}^*$ is called the α-minimum spanning tree if*

$$\min\{\bar{z} \mid \mathrm{Cr}\{z(\boldsymbol{x}^*,\boldsymbol{\xi}) \le \bar{z}\} \ge \alpha\} \le \min\{\bar{z} \mid \mathrm{Cr}\{z(\boldsymbol{x},\boldsymbol{\xi}) \le \bar{z}\} \ge \alpha\} \tag{8}$$

for all spanning tree $\boldsymbol{x} \in X$, where α is predetermined confidence level and $\min\{\bar{z} \mid \mathrm{Cr}\{z(\boldsymbol{x}^*,\boldsymbol{\xi}) \le \bar{z}\} \ge \alpha\}$ is called the α-minimum cost.

Sometimes, the decision maker may provide a cost supremum $\bar{z}$ and hope the credibility of the cost not exceeding $\bar{z}$ is as maximized as possible. For this case, we propose the concept of the most minimum spanning tree (MMST) as follows:

Definition 5 *A spanning tree $\boldsymbol{x}^*$ is called the most minimum spanning tree if*

$$\mathrm{Cr}\left\{z(\boldsymbol{x}^*,\boldsymbol{\xi}) \le \bar{z}\right\} \ge \mathrm{Cr}\left\{z(\boldsymbol{x},\boldsymbol{\xi}) \le \bar{z}\right\} \tag{9}$$

for all spanning tree $\boldsymbol{x} \in X$, where $\bar{z}$ is a predetermined cost supremum.

4 Fuzzy Multi-Criteria Minimum Spanning Tree Model

In this section, we use the concepts of EMST, α-MST and MMST as decision criteria, and formulate the FMCMST problem as expected minimum spanning tree model, α-minimum spanning tree model and the most minimum spanning tree model, respectively.

Fuzzy expected value model, which was presented by Liu and Liu [15], is to optimize the expected objective subject to some constraints. In order to find the EMST with fuzzy costs, distances, and so on, we have the following expected value model:

$$\begin{cases} \min \ \left\{ \mathrm{E}\left[\sum_{i=1}^{m} \xi_{1i} x_i\right], \mathrm{E}\left[\sum_{i=1}^{m} \xi_{2i} x_i\right], \cdots, \mathrm{E}\left[\sum_{i=1}^{m} \xi_{pi} x_i\right] \right\} \\ \text{subject to:} \\ \qquad x_i = 0 \text{ or } 1 \\ \qquad \boldsymbol{x} \in X. \end{cases} \tag{10}$$

Chance-constrained programming offers us a powerful means for modelling stochastic decision systems [3] and fuzzy decision systems [10][11][12]. The essential idea of chance-constrained programming is to optimize the critical value of the fuzzy objective with certain confidence level subject to some chance constraints. In order to find the α-MST, where α is a confidence level provided by the decision maker, we propose the following chance-constrained programming model:

$$\begin{cases} \min \ \{\bar{z_1}, \bar{z_2}, \cdots, \bar{z_p}\} \\ \text{subject to:} \\ \qquad \mathrm{Cr}\left\{ \sum_{i=1}^{m} \xi_{li} x_i \le \bar{z_l} \right\} \ge \alpha, l = 1, 2, \cdots, p \\ \qquad x_i = 0 \text{ or } 1 \\ \qquad \boldsymbol{x} \in X. \end{cases} \tag{11}$$

Sometimes the decision maker hopes to maximize the chance functions of some events (i.e. the credibility satisfying these fuzzy events). In order to model this type of fuzzy decision system, Liu [13][14] provided one type of fuzzy programming model: dependent-chance programming, in which the underlying philosophy is based on selecting the decision with maximal chance to meet the fuzzy event. Now let us model the FMCMST problem by dependent-chance programming. Suppose that the decision maker sets a cost supremum $\bar{z}_l$, and hopes to find the MMST, we have the dependent-chance programming model as follows:

$$\begin{cases} \max \ \left\{ \mathrm{Cr}\left\{ \sum_{i=1}^{m} \xi_{1i} x_i \le \bar{z_1} \right\}, \mathrm{Cr}\left\{ \sum_{i=1}^{m} \xi_{2i} x_i \le \bar{z_2} \right\}, \cdots, \mathrm{Cr}\left\{ \sum_{i=1}^{m} \xi_{pi} x_i \le \bar{z_p} \right\} \right\} \\ \text{subject to:} \\ \qquad x_i = 0 \text{ or } 1 \\ \qquad \boldsymbol{x} \in X. \end{cases} \tag{12}$$

5 Crisp Equivalents

In this section, based on the credibility theory, we propose the crisp equivalents of the proposed FMCMST models under some assumptions.

Theorem 1 *Let $\xi_{li} (i = 1, 2, \cdots, m, l = 1, 2, \cdots, p)$ be independent fuzzy variables. Then the crisp equivalent of the FMCMST model (10) is*

$$\begin{cases} \min \ \left\{ \sum_{i=1}^{m} \mathrm{E}\left[\xi_{1i}\right] x_i, \sum_{i=1}^{m} \mathrm{E}\left[\xi_{2i}\right] x_i, \cdots, \sum_{i=1}^{m} \mathrm{E}\left[\xi_{pi}\right] x_i \right\} \\ \text{subject to:} \\ \qquad x_i = 0 \ \text{or} \ 1 \\ \qquad \boldsymbol{x} \in X. \end{cases} \tag{13}$$

Now, we suppose that the weights such as distances, costs are independent triangular fuzzy numbers, and give the crisp equivalents of FMCMST models (11) and (12), respectively.

By *triangular fuzzy variables* we mean the fuzzy variables fully determined by triplet (r_1, r_2, r_3) of crisp numbers with $r_1 < r_2 < r_3$, whose membership functions can be denoted by

$$\mu(x) = \begin{cases} \dfrac{x - r_1}{r_2 - r_1}, & \text{if } r_1 \le x \le r_2 \\ \dfrac{x - r_3}{r_2 - r_3}, & \text{if } r_2 \le x \le r_3 \\ 0, & \text{otherwise.} \end{cases} \tag{14}$$

Let $\xi = (r_1, r_2, r_3)$ and $\eta = (q_1, q_2, q_3)$ be two triangular fuzzy variables, and a, b two nonnegative numbers. Then we have that

$$a\xi + b\eta = (ar_1 + bq_1, ar_2 + bq_2, ar_3 + bq_3). \tag{15}$$

Let $\xi = (r_1, r_2, r_3)$ be a triangular fuzzy variable. Then the credibility distribution of ξ is continuous and defined by

$$\mathrm{Cr}\{\xi \le \bar{z}\} = \begin{cases} 1, & \text{if } r_3 \le \bar{z} \\ \dfrac{\bar{z} + r_3 - 2r_2}{2(r_3 - r_2)}, & \text{if } r_2 \le \bar{z} \le r_3 \\ \dfrac{\bar{z} - r_1}{2(r_2 - r_1)}, & \text{if } r_1 \le \bar{z} \le r_2 \\ 0, & \text{otherwise.} \end{cases} \tag{16}$$

Let $\xi = (r_1, r_2, r_3)$ be a triangular fuzzy variable, and α a given confidence level. Then we have

$$\text{(a) when } \ \alpha \le 1/2, \mathrm{Cr}\{\xi \le \bar{z}\} \ge \alpha$$

$$\text{if and only if } \ (1 - 2\alpha)r_1 + 2\alpha r_2 \le \bar{z}; \tag{17}$$

$$\text{(b) when } \ \alpha > 1/2, \mathrm{Cr}\{\xi \le \bar{z}\} \ge \alpha$$

$$\text{if and only if } \ (2 - 2\alpha)r_2 + (2\alpha - 1)r_3 \le \bar{z}. \tag{18}$$

Theorem 2 *Let $\xi_{li} = (s_{li1}, s_{li2}, s_{li3})$, $i = 1, 2, \cdots, m$, $l = 1, 2, \cdots, p$ be independent triangular fuzzy variables. If $\alpha > 0.5$, then the crisp equivalent of the FMCMST model (11) is given by*

$$\begin{cases} \min \ \left\{ (2 - 2\alpha) \left(\sum_{i=1}^{m} s_{li2} x_i \right) + (2\alpha - 1) \left(\sum_{i=1}^{m} s_{li3} x_i \right), l = 1, 2, \cdots, p \right\} \\ \text{subject to :} \\ \qquad x_i = 0 \text{ or } 1 \\ \qquad \boldsymbol{x} \in X. \end{cases} \tag{19}$$

Proof. Since $x_i \ge 0$ for $i = 1, 2, \cdots, m$, it follows from (15) that the cost function

$$\sum_{i=1}^{m} \xi_{li} x_i$$

is also a triangular fuzzy variable, and determined by the triplet

$$(g_{l1}(\boldsymbol{x}), g_{l2}(\boldsymbol{x}), g_{l3}(\boldsymbol{x})) = (\sum_{i=1}^{m} s_{li1}x_i, \sum_{i=1}^{m} s_{li2}x_i, \sum_{i=1}^{m} s_{li3}x_i). \tag{20}$$

Then it follows from (18) that the chance constraint

$$\mathrm{Cr}\left\{\sum_{i=1}^{m} \xi_{li}x_i \le \bar{z}_l\right\} \ge \alpha$$

is equivalent to

$$(2-2\alpha)\left(\sum_{i=1}^{m} s_{li2}x_i\right) + (2\alpha-1)\left(\sum_{i=1}^{m} s_{li3}x_i\right) \le \bar{z}_l. \tag{21}$$

That is, the FMCMST model (11) is equivalent to model (19).

Theorem 3 *Let* $\xi_{li} = (s_{li1}, s_{li2}, s_{li3})$, $i = 1, 2, \cdots, m$, $l = 1, 2, \cdots, p$ *be independent triangular fuzzy variables. Then the crisp equivalent of the fuzzy dependent-chance programming (12) is given by*

$$\begin{cases} \max \ \ \{f_l(\boldsymbol{x}), l = 1, 2, \cdots, p\} \\ \text{subject to:} \\ \qquad x_i = 0 \ \mathit{or} \ 1 \\ \qquad \boldsymbol{x} \in X, \end{cases} \tag{22}$$

where $f_l(\boldsymbol{x})$ *is a real function defined by*

$$f_l(\boldsymbol{x}) = \begin{cases} 1, & \text{if } g_{l3}(\boldsymbol{x}) \le \bar{z}_l \\ \dfrac{\bar{z}_l + g_{l3}(\boldsymbol{x}) - 2g_{l2}(\boldsymbol{x})}{2(g_{l3}(\boldsymbol{x}) - g_{l2}(\boldsymbol{x}))}, & \text{if } g_{l2}(\boldsymbol{x}) \le \bar{z}_l \le g_{l3}(\boldsymbol{x}) \\ \dfrac{\bar{z}_l - g_{l1}(\boldsymbol{x})}{2(g_{l2}(\boldsymbol{x}) - g_{l1}(\boldsymbol{x}))}, & \text{if } g_{l1}(\boldsymbol{x}) \le \bar{z}_l \le g_{l2}(\boldsymbol{x}) \\ 0, & \text{otherwise} \end{cases} \tag{23}$$

and $g_{li}(\boldsymbol{x})$, $i = 1, 2, 3$ *are defined by (20).*

Proof. Since $x_i \ge 0$ for $i = 1, 2, \cdots, m$, it follows from (15) that the cost function

$$\sum_{i=1}^{m} \xi_{li}x_i$$

is also a triangular fuzzy number, and determined by the triplet $(g_{l1}(\boldsymbol{x}), g_{l2}(\boldsymbol{x}), g_{l3}(\boldsymbol{x}))$ defined in equation (20). Then it follows from (5) that the chance function

$$\mathrm{Cr}\left\{\sum_{i=1}^{m} \xi_{li}x_i \le \bar{z}_l\right\}$$

is equivalent to the real function $f_l(\boldsymbol{x})$ defined by equation (23). That is, the FMCMST model (12) is equivalent to model (22).

All the conclusions above can be extended for trapezoidal fuzzy variables similarly.

6 Hybrid Intelligent Algorithm

Due to the complexity of the problems, we design a hybrid intelligent algorithm integrating fuzzy simulation and genetic algorithm to solve them.

Genetic algorithm was developed by Holland [7] in 1975. It is considered as one of the most efficient intelligent algorithms.

Zhou and Gen [20] showed that genetic algorithm can deal with the MCMST problem effectively. As an extension of MCMST problem, the FMCMST problem is more difficult to solve. In the following, we will design a fuzzy simulation-based genetic algorithm for solving the proposed FMCMST models as well as their crisp equivalents.

6.1 Chromosome Representation

In the FMCMST problem, a chromosome should represent a spanning tree. One of the classical theorems in graphical enumeration is Cayley's theorem. It shows that there are n^{n-2} distinct labelled trees on a complete graph with n vertices. Prüfer proved it by establishing one-to-one correspondence between such trees and the set of all permutation of $n-2$ digits. And this permutation is known as the Prüfer number. Then we use the Prüfer number as chromosome, which uniquely represents a spanning tree. The reader can read the book [6] by Gen and Cheng for details.

For any tree in a complete graph, there are at least two leaf vertices. Leaf vertex means that there is only one edge connected with it. Based on this observation, Zhou and Gen [20] constructed the following encoding and decoding procedure.

Encoding Procedure

Step 1. Let vertex j be the smallest labelled leaf vertex in a labelled tree.

Step 2. Set k to the first digit in the permutation if vertex k is incident to vertex j.

Step 3. Remove vertex j and the edge from j to k, we have a tree with $n-1$ vertices.

Step 4. Repeat above steps until one edge is left and produce the Prüfer number or permutation with $n-2$ digits in order.

Decoding Procedure

Step 1. Let P be the original Prüfer number, and let $\overline{P}$ be the set of all vertices not included in P.

Step 2. Let j be the vertex with the smallest label in $\overline{P}$, and let k be the leftmost digit of P. Add the edge from j to k into the tree. Remove j from $\overline{P}$ and k from P. If k does not occur anywhere in the remainder of P, put it into $\overline{P}$. Repeat the process until no digits are left in P.

Step 3. If no digits remain in P, there are exactly two vertices, r and s, in $\overline{P}$. Add edge from r to s into the tree and form a tree with $n-1$ edges.

6.2 Fuzzy Simulation

If the fuzzy chance-constrained programming can be converted to its deterministic equivalent, then it is easy for us to compute the objective function. Otherwise, we may employ the fuzzy simulation technique. It has been discussed in detail by Liu [16] that how to compute the credibility, critical value and expected value defined by

$$\mathrm{Cr}\left\{\sum_{i=1}^{m}\xi_{li}x_i \le \bar{z}_l\right\} \tag{24}$$

$$\inf\left\{\bar{z}_l \mid \mathrm{Cr}\left\{\sum_{i=1}^{m}\xi_{li}x_i \le \bar{z}_l\right\} \ge \alpha\right\} \tag{25}$$

and

$$\mathrm{E}\left[\sum_{i=1}^{m} \xi_{li} x_i\right], \tag{26}$$

respectively. The procedures for computing them are given as follows:

Fuzzy Simulation for Credibility:

Step 1. Randomly generate u_{lik} from the ε-level set of ξ_{li} and write $\nu_{lk} = \mu(u_{lik})$, $l = 1, 2, \cdots, p; i = 1, 2, \cdots, m$, respectively, where $k = 1, 2, \cdots, N$ and ε is a sufficiently small positive number.

Step 2. Return $L_l(\bar{C})$ via the following estimation formula (27)

$$L_l(r) = \frac{1}{2}\left(\max_{1\le k\le N}\left\{\nu_{lk} \mid C_{lk}(\boldsymbol{x}) \le r\right\} + \min_{1\le k\le N}\left\{1 - \nu_{lk} \mid C_{lk}(\boldsymbol{x}) > r\right\}\right) \tag{27}$$

where

$$C_{lk}(\boldsymbol{x}) = \sum_{i=1}^{m} u_{lik} x_i. \tag{28}$$

Fuzzy Simulation for α-Critical Value:

Step 1. Randomly generate u_{lik} from the ε-level set of ξ_{li}, $l = 1, 2, \cdots, p; i = 1, 2, \cdots, m$, respectively, where $k = 1, 2, \cdots, N$ and ε is a sufficiently small positive number.

Step 2. Find the minimal value r such that $L_l(r) \ge \alpha$ holds, where $L_l(r)$ is defined by (27).

Step 3. Return r.

Fuzzy Simulation for Expected Value:

Step 1. Set $e = 0$.

Step 2. Randomly generate u_{lik} from the ε-level set of ξ_{li}, $l = 1, 2, \cdots, p; i = 1, 2, \cdots, m$, respectively, where $k = 1, 2, \cdots, N$ and ε is a sufficiently small positive number.

Step 3. Set $b_{l1} = C_{l1}(\boldsymbol{x}) \wedge \cdots \wedge C_{lN}(\boldsymbol{x})$ and $b_{l2} = C_{l1}(\boldsymbol{x}) \vee \cdots \vee C_{lN}(\boldsymbol{x})$.

Step 4. Randomly generate b_l from $[b_{l1}, b_{l2}]$.

Step 5. If $b_l \ge 0$, then $e \leftarrow e + \mathrm{Cr}\{C_{lk}(\boldsymbol{x}) \le b_l\}$.

Step 6. If $b_l < 0$, then $e \leftarrow e - \mathrm{Cr}\{C_{lk}(\boldsymbol{x}) \ge b_l\}$.

Step 7. Repeat the third to fifth steps for N times.

Step 8. Return $\mathrm{E}[C_{lk}(\boldsymbol{x})] = b_{l1} \vee 0 + b_{l2} \wedge 0 + e \cdot (b_{l2} - b_{l1})/N$.

6.3 Crossover and Mutation Operation

Since a Prüfer number can always represent a labelled tree, we select a simple way for crossover and mutation operation. For two chromosomes to crossover, we just exchange their digits at randomly selected positions. And for a chromosome to mutate, randomly select a position and randomly generate an integer between 1 and n including 1 and n to replace the original one.

6.4 Evaluation and Selection Process

In our genetic algorithm approach for FMCMST problem, the evaluation performs the following operations: (i) decoding all the chromosomes and calculating their expected cost, α-critical cost and chance function; (ii) assigning each chromosome a fitness by a rank-based method according to its objective value. Then in the selection process, by spinning the roulette wheel *pop_size* times, we get a new population to go further.

6.5 Hybrid Intelligent Algorithm

We embed fuzzy simulation into genetic algorithm to produce a hybrid intelligent algorithm. The procedure is as follows:

Genetic algorithm procedure

Step 1. Initialize *pop_size* chromosomes randomly.

Step 2. Update the chromosomes by crossover and mutation operations.

Step 3. Calculate the objective values for all chromosomes.

Step 4. Compute the fitness of each chromosome according to the objective values.

Step 5. Select the chromosomes by spinning the roulette wheel.

Step 6. Repeat the second to fifth steps for a given number of cycles.

Step 7. Report the best chromosome as the optimal solution.

7 Numerical Example

In this section, we give a numerical example that is performed on a personal computer to illustrate the effectiveness of the simulation-based genetic algorithm.
Example. Consider an FMCMST problem with 6 vertices. For a complete graph, we label its 6 vertices with integers $1, 2, \cdots, 6$, respectively. Then it has $C_6^2 = 15$ edges. Two weight attributes defined on each edge are independent triangular fuzzy numbers or trapezoidal fuzzy numbers. And the preference values are $(0.3, 0.7)$.

In order to find the EMST and 0.90-MST and MMST (cost supremum is set as 28 by the decision maker), we formulate the FMCMST problem as expected minimum spanning tree model (10), α-minimum spanning tree model (11) and the most minimum spanning tree model (12), respectively. Then, based on Theorem 1, 2 and 3, we convert them to their crisp equivalents (13), (19) and (22), respectively. After a run of the genetic algorithm with $pop\_size = 30$, we get that the EMST, 0.90-MST and MMST are the spanning trees with Prüfer numbers $(2, 5, 1, 2)$, $(2, 2, 1, 2)$ and $(2, 2, 1, 2)$, respectively. The corresponding expected minimum cost, 0.90-minimum cost and most credibility are 17.8250, 28.2 and 0.9192, respectively. Now, we convert the Prüfer number $(2, 2, 1, 2)$ to a spanning tree by the decoding procedure as a demo.

1. Let $P = (2, 2, 1, 2)$ and $\overline{P} = \{3, 4, 5, 6\}$.

2. Remove the smallest integer 3 from $\overline{P}$, and the leftmost integer 2 from P. Add the edge from 3 to 2 to the tree. Since 2 occurs at other places of P, we have $P = (2, 1, 2)$ and $\overline{P} = \{4, 5, 6\}$.

3. Remove the smallest integer 4 from $\overline{P}$, and the leftmost integer 2 from P. Add the edge from 4 to 2 to the tree. Since 2 occurs at other places of P, we have $P = (1, 2)$ and $\overline{P} = \{5, 6\}$.

4. Remove the smallest integer 5 from $\overline{P}$, and the leftmost integer 1 from P. Add the edge from 5 to 1 to the tree. Since 1 does not occur at other places of P, we have $P = (2)$, $\overline{P} = \{1, 6\}$.

5. Remove the smallest integer 1 from $\overline{P}$, and the leftmost integer 2 from P. Add the edge from 1 to 2 to the tree. Now we have $\overline{P} = \{2, 6\}$, $P = \emptyset$.

6. Add the edge from vertex 2 to 6 to the tree, and we get a tree with five edges, i.e., a spanning tree in Figure 1.

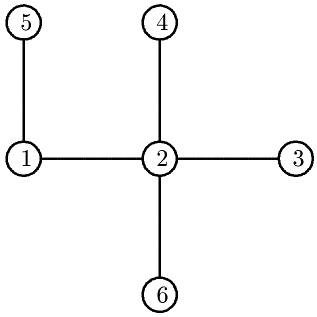

Figure 1: The spanning tree associated with Prüfer number (2,2,1,2)

8 Conclusions

In this paper, the FMCMST problem was formulated as expected value model, chance-constrained programming and dependence-chance programming. Their crisp equivalent models were also proposed based on the credibility theory. Moreover, a hybrid intelligent algorithm approach was proposed for solving the proposed FMCMST models as well as their crisp equivalents. A numerical example was also provided for illustrating the effectiveness of the genetic algorithm.

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